Thanks to the Simons Institute
for the Theory of Computing

SIMONS INSTITUTE
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https://simons.berkeley.edu/programs/logic-algorithms-database-theory-ai

DATALAB
@Northeastern

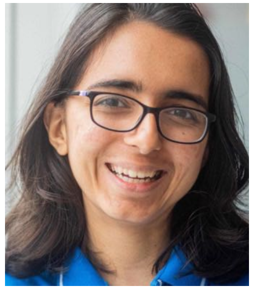
Is Integer Linear Programming All You Need for Deletion Propagation?

A unified and practical approach to Generalized Deletion Propagation

VLDB 2025 (London, Sept 4, 2025)

https://northeastern-datalab.github.io/unified-reverse-data-management/

Neha Makhija
Northeastern University
(UMass Amherst from Sept 2025)

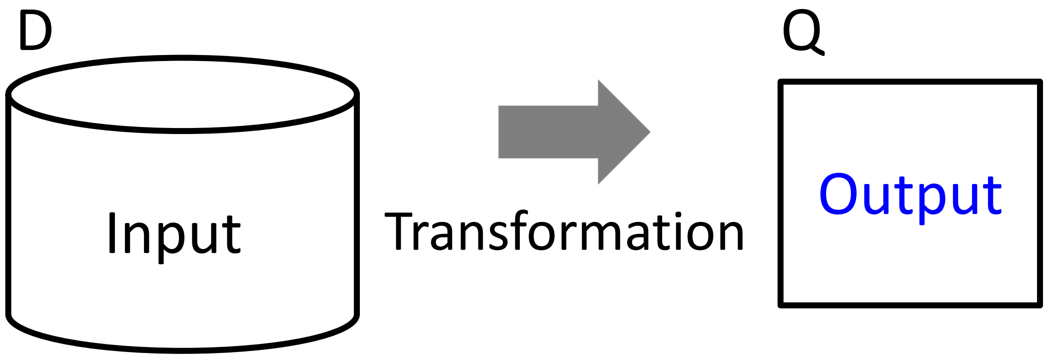


Wolfgang Gatterbauer
Northeastern University

Data Management

QUERY EVALUATION:

A transformation of the input to the output



D

Input

Transformation

Q

Output

Reverse Data Management (RDM)

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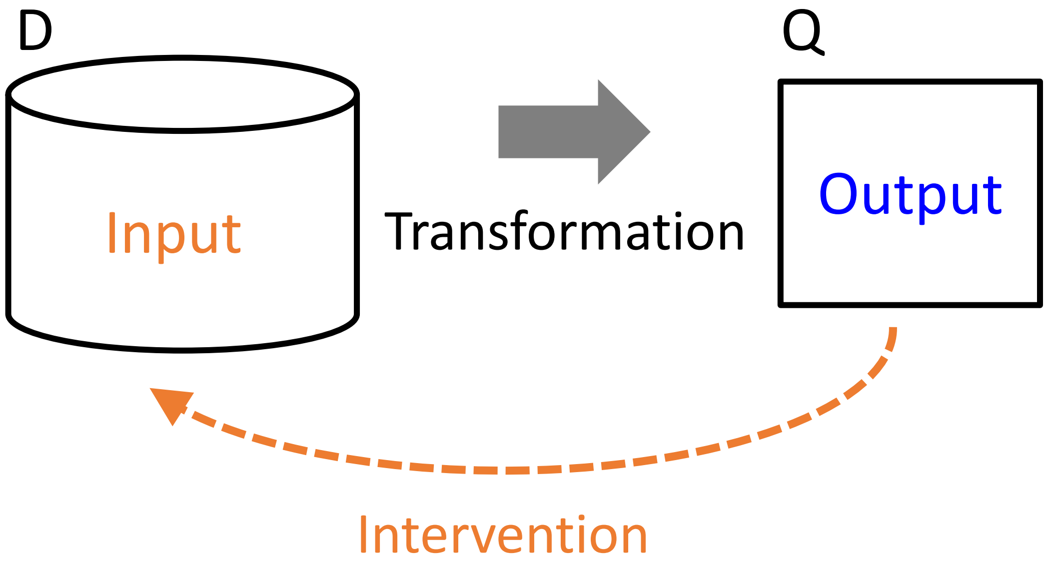

REVERSE DATA MANAGEMENT (RDM):

What are the required changes to the input, in order to achieve a desired output?

Meliou, Gatterbauer, Suciu. Reverse Data Management. VLDB 2011 Vision track. https://doi.org/10.14778/3402755.3402803

Reverse Data Management (RDM): 2 types of Explanations

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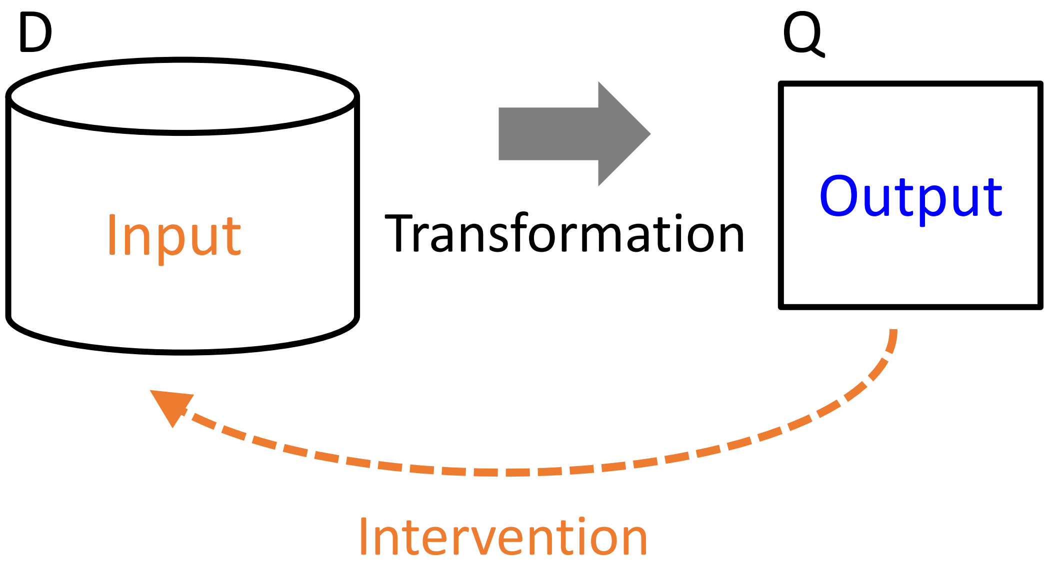
A transformation of the input to the output



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1. Contrastive (counterfactual):

2. Abductive (factual):

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Marques-Silva. Logic-Based Explainability in Machine Learning. Reasoning Web. Causality, Explanations and Declarative Knowledge. 2023. https://doi.org/10.1007/978-3-031-31414-8_2

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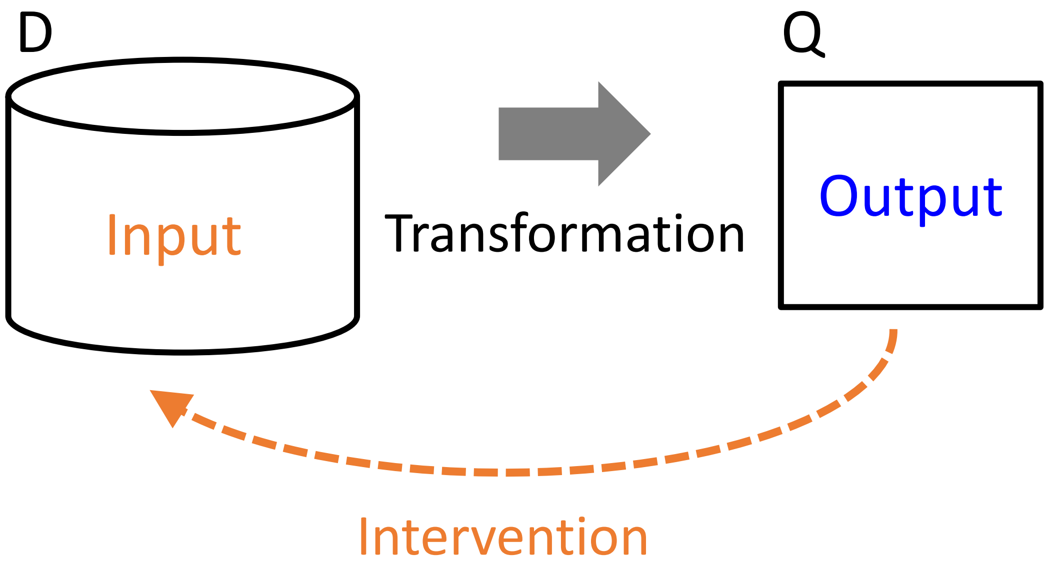

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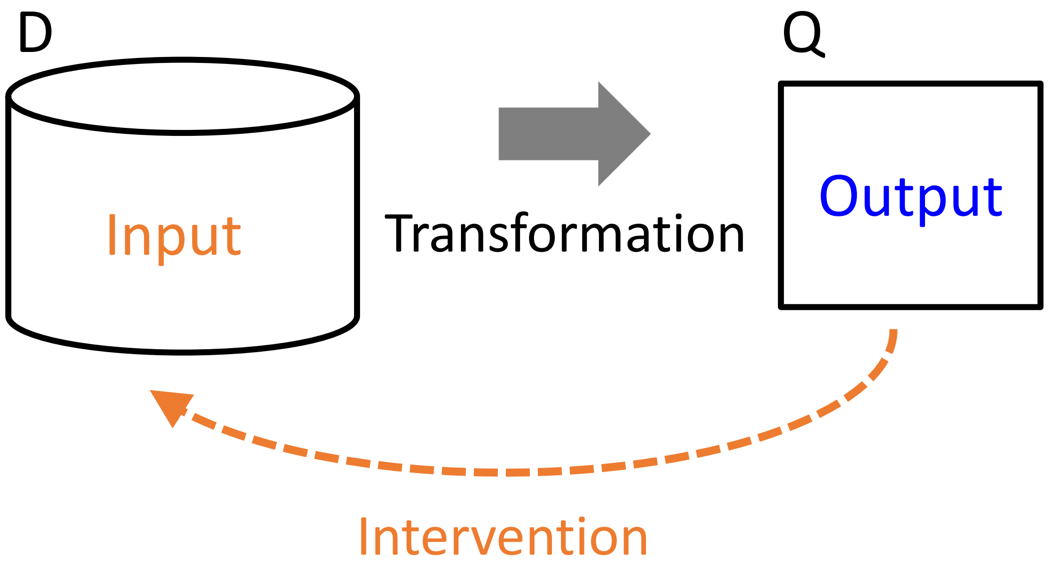


REVERSE DATA MANAGEMENT (RDM):

What are the required changes to the input, in order to achieve a desired output?

Two types of explanations in RDM, naming used from Explainable AI (XAI):

1. Contrastive (counterfactual): set of tuples (features) of min size that are sufficient to change an output (prediction)

= min deletions of tuples

(e.g., resilience, deletion propagation)

2. Abductive (factual): set of tuples (features) of min size that are sufficient for ensuring a certain output (prediction)

= max deletions of tuples

(e.g., smallest witness problem)

= DELETION PROPAGATION

Meliou, Gatterbauer, Suciu. Reverse Data Management. VLDB 2011 Vision track. https://doi.org/10.14778/3402755.3402803

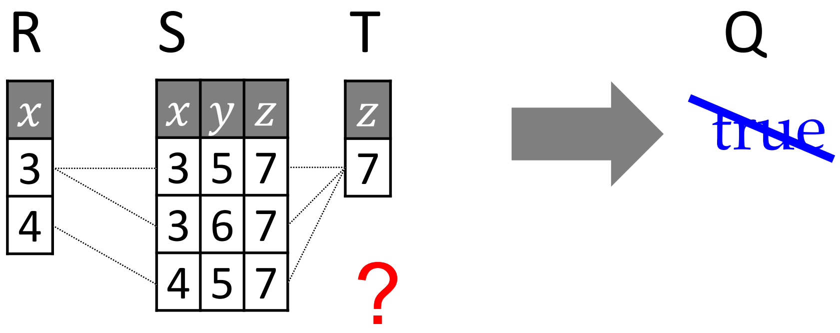
Marques-Silva. Logic-Based Explainability in Machine Learning. Reasoning Web. Causality, Explanations and Declarative Knowledge. 2023. https://doi.org/10.1007/978-3-031-31414-8_2

Example Reverse Data Management Problems

Q:-R(x),S(x,y,z),T(z)

1. Resilience

Delete min number of tuples to make Boolean Q false



R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
7

```
select exists(
  select 1
  from R, S, T
  where R.x=S.x
  and S.z=T.z)
```

Example Reverse Data Management Problems

Q:-R(x),S(x,y,z),T(z)

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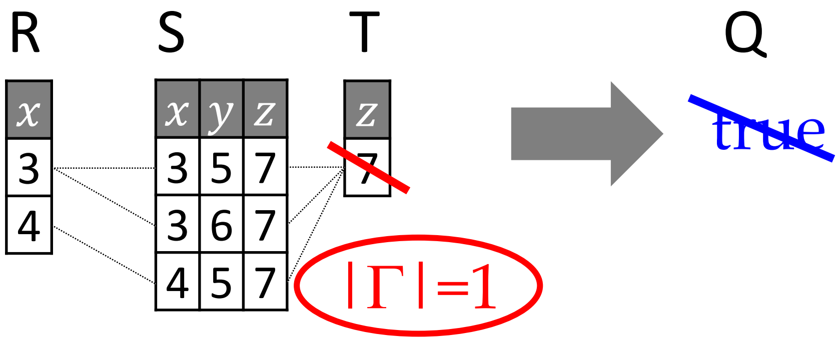

R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

Q

~~true~~

$|\Gamma|=1$

```
select exists(
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  from R, S, T
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Freire, Gatterbauer, Immerman, Meliou. The complexity of resilience and responsibility for self-join-free conjunctive queries, VLDB 2015. https://doi.org/10.14778/2850583.2850592

Makhija, Gatterbauer. A Unified Approach for Resilience and Causal Responsibility with Integer Linear Programming (ILP) and LP Relaxations, SIGMOD 2024. https://doi.org/10.1145/3626715

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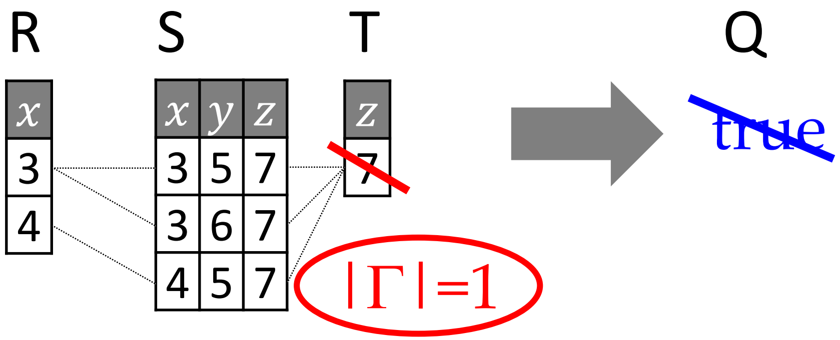

R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

Q

~~true~~

```
select exists(
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  from R, S, T
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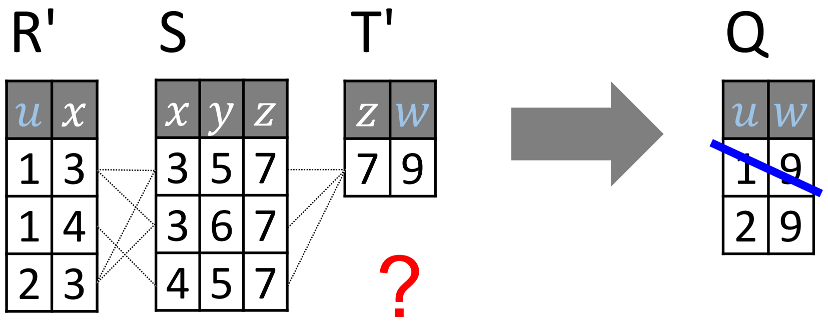
```

Q(u,w):-R'(u,x),S(x,y,z),T'(z,w)

2. Source side-effects (deletion propagation)

Delete min number of tuples to delete an output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
7	9

Q

u	w
~~1~~	~~9~~
2	9

```
select R'.u, T'.w
from R', S, T'
where R'.x=S.x
and S.z=T'.z
```

Dayal, Bernstein. On the correct translation of update operations on relational views, TODS 1982, https://doi.org/10.1145/319732.319740
Buneman, Khanna, Tan. On propagation of deletions and annotations through views, PODS 2002, https://doi.org/10.1145/543613.543633

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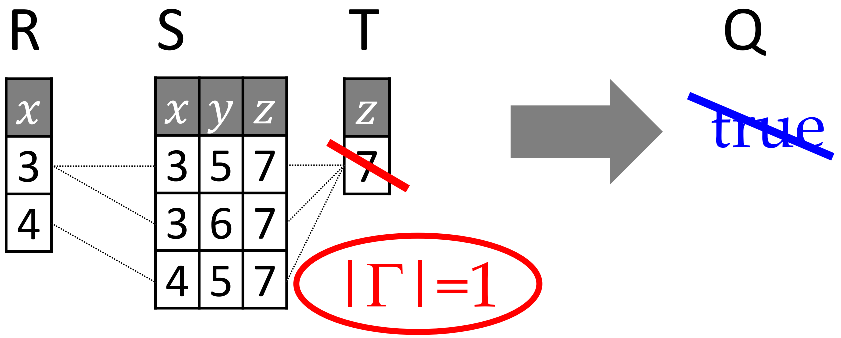


R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

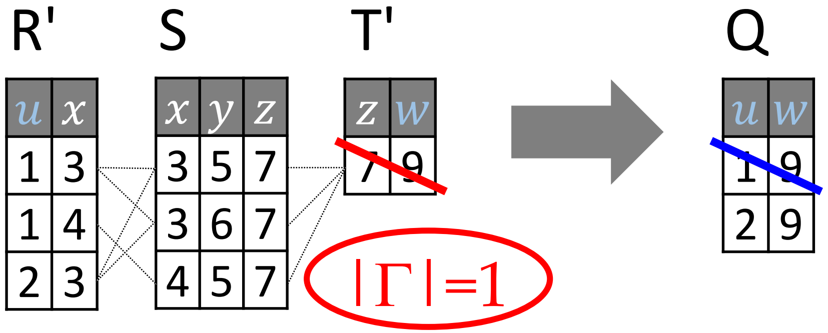
Q

~~true~~

$|\Gamma|=1$

2. Source side-effects (deletion propagation)

Delete min number of tuples to delete an output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

Q

u	w
~~1~~	~~9~~
2	9

$|\Gamma|=1$

basically identical

A long open problem: for which conjunctive queries is resilience in PTIME? (and for which NP-complete?). Only a partial classification known today

Dayal, Bernstein. On the correct translation of update operations on relational views, TODS 1982, https://doi.org/10.1145/319732.319740
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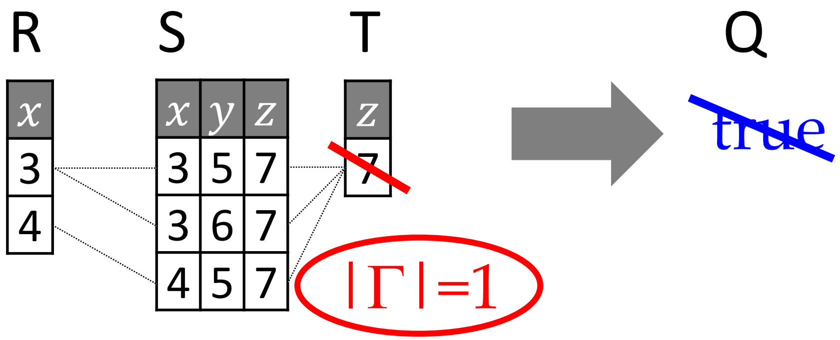


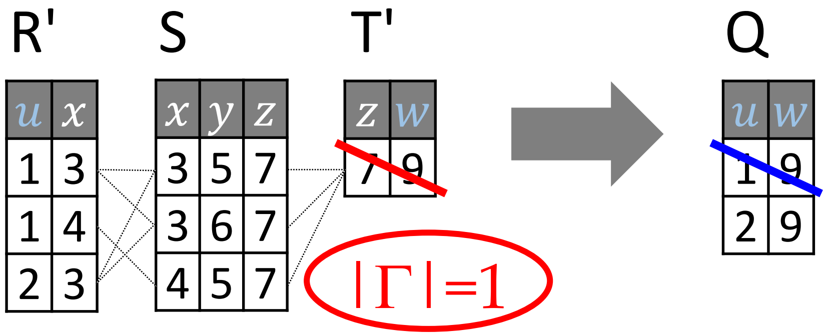
x
3
4

x	y	z
3	5	7
3	6	7
4	5	7

z
~~7~~

2/3. (Aggregated) Source side-effects (d.p.)

Delete min number of tuples to delete ≥k output tuples



u	x
1	3
1	4
2	3

x	y	z
3	5	7
3	6	7
4	5	7

z	w
~~7~~	~~9~~

u	w
~~1~~	~~9~~
2	9

Different tractability results!

Hu, Sun, Patwa, Panigrahi, Roy. Aggregated Deletion Propagation for Counting Conjunctive Query Answers. PVLDB 2020. https://doi.org/10.14778/3425879.3425892

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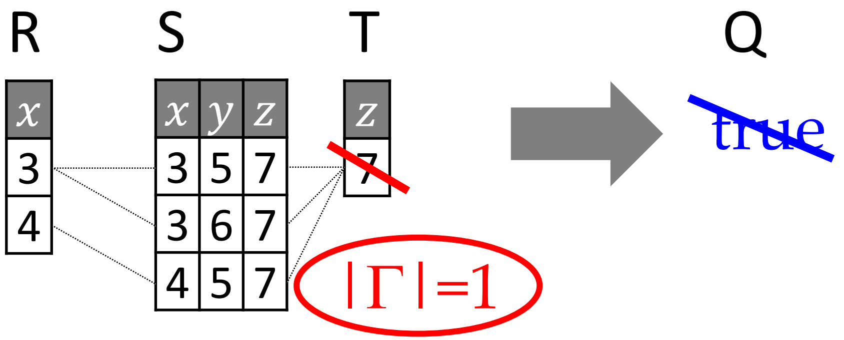

R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

Q

~~true~~

$|\Gamma|=1$

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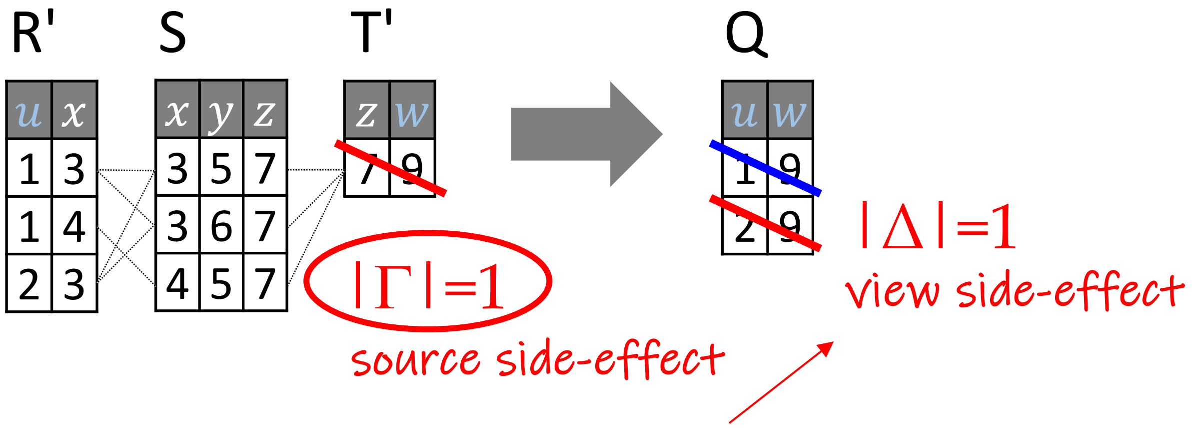

R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

Q

u	w
~~1~~	~~9~~
~~2~~	~~9~~

$|\Delta|=1$

view side-effect

$|\Gamma|=1$

source side-effect

Oops, we deleted more output tuples than needed.

That is not a concern when minimizing source-side effects.
But when minimizing view-side effects!

Hu, Sun, Patwa, Panigrahi, Roy. Aggregated Deletion Propagation for Counting Conjunctive Query Answers. PVLDB 2020. https://doi.org/10.14778/3425879.3425892

Example Reverse Data Management Problems

$Q\text{:-}R(x),S(x,y,z),T(z)$

$Q(u,w)\text{:-}R'(u,x),S(x,y,z),T'(z,w)$

1. Resilience

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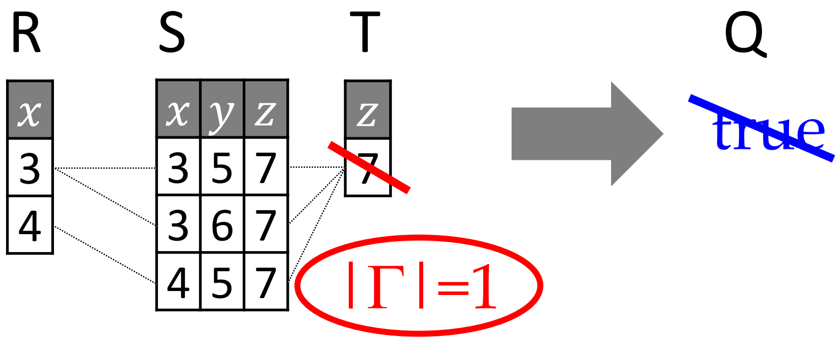

R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

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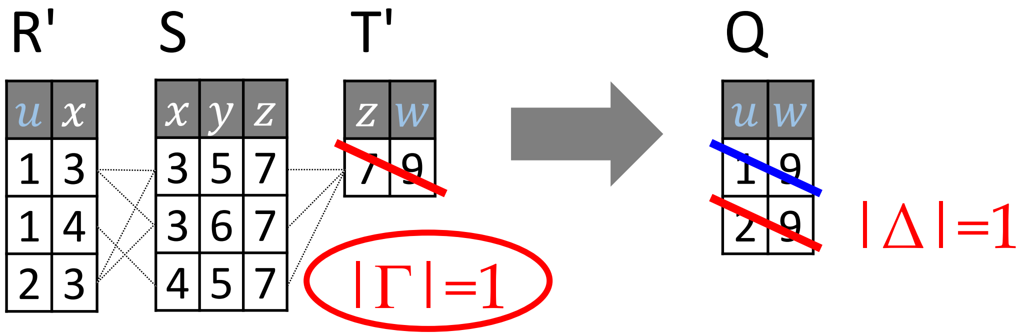

R'

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1	3
1	4
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S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

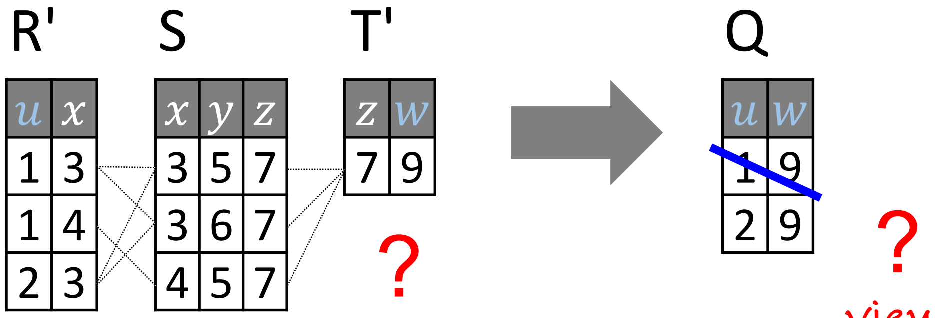
Q

u	w
~~1~~	~~9~~
~~2~~	~~9~~

source side-effect

4. View side-effects (deletion propagation)

Delete tuples in order to delete an output tuple, while minimizing the other output tuples deleted



R'

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T'

z	w
7	9

Q

u	w
~~1~~	~~9~~
2	9

view side-effect

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Kimelfeld, Vondrak, Williams. Maximizing conjunctive views in deletion propagation, PODS 2011, https://doi.org/10.1145/1989284.1989308

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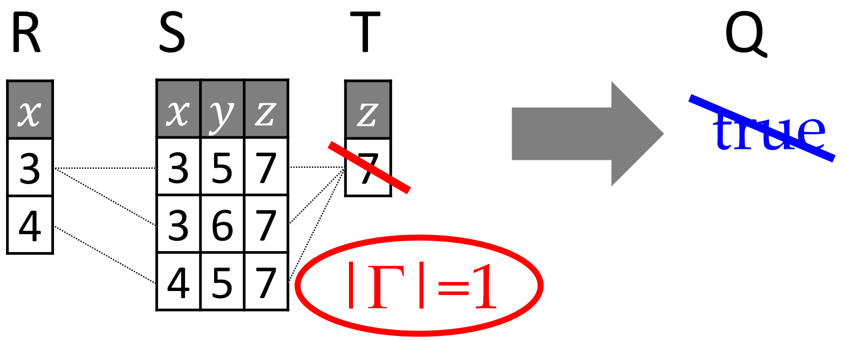


R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

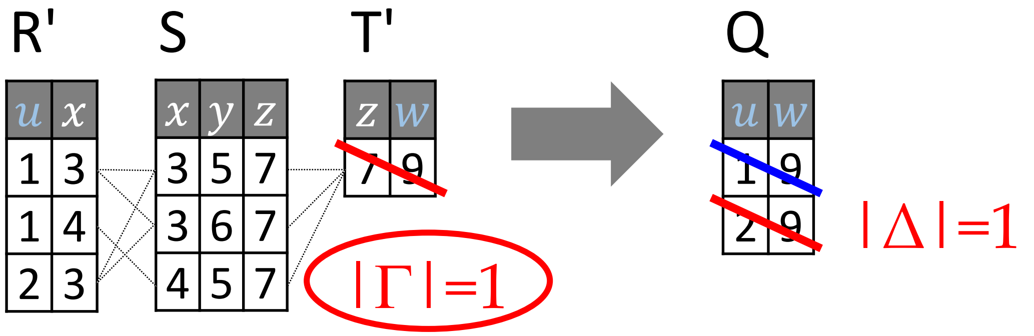
z
~~7~~

Q

~~true~~

2/3. (Aggregated) Source side-effects (d.p.)

Delete min number of tuples to delete ≥k output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

Q

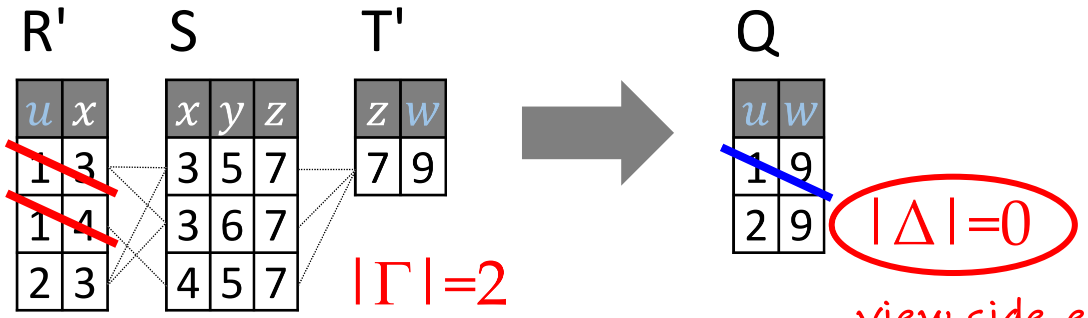
u	w
~~1~~	~~9~~
~~2~~	~~9~~

source side-effect

4. View side-effects (deletion propagation)

Delete tuples in order to delete an output tuple, while minimizing the other output tuples deleted

Again: Different tractability results!



R'

u	x
~~1~~	~~3~~
~~1~~	~~4~~
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
7	9

Q

u	w
~~1~~	~~9~~
2	9

view side-effect

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Example Reverse Data Management Problems

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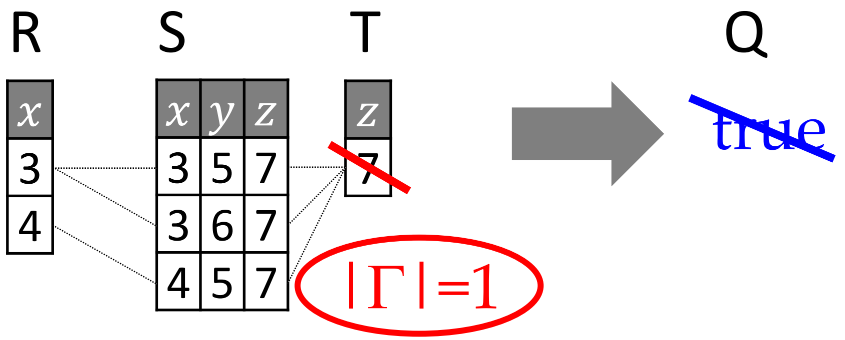
1. Resilience

Delete min number of tuples to make Boolean Q false



R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

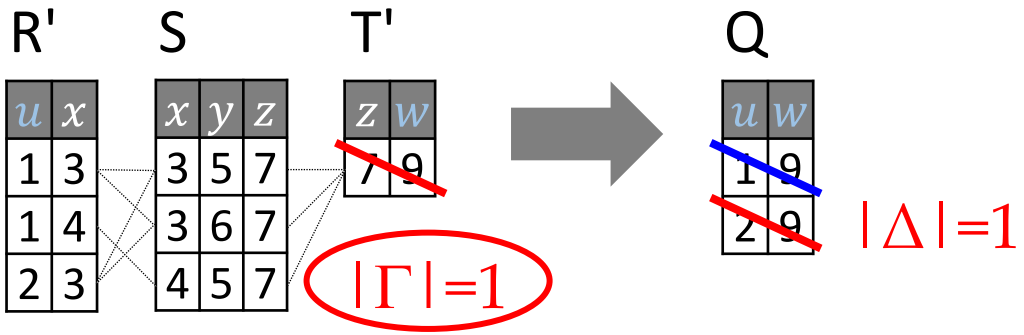
Q

~~true~~

$|\Gamma|=1$

2/3. (Aggregated) Source side-effects (d.p.)

Delete min number of tuples to delete ≥k output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

Q

u	w
~~1~~	~~9~~
~~2~~	~~9~~

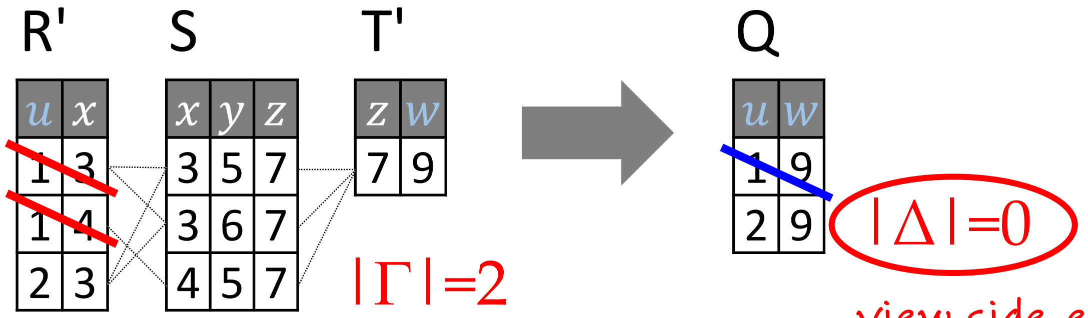
$|\Gamma|=1$

$|\Delta|=1$

source side-effect

4/5. (Aggregated) View side-effects (d.p.)

Delete tuples in order to delete ≥k output tuple,
while minimizing the other output tuples deleted



R'

u	x
~~1~~	~~3~~
~~1~~	~~4~~
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
7	9

Q

u	w
~~1~~	~~9~~
2	9

$|\Gamma|=2$

$|\Delta|=0$

view side-effect

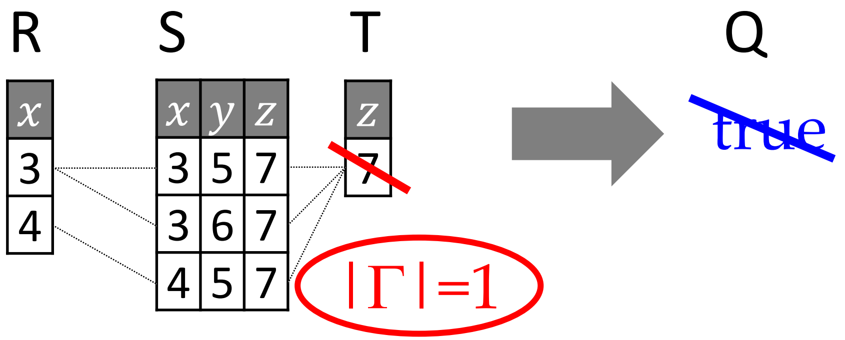
No prior work yet

Example Reverse Data Management Problems

Q:-R(x),S(x,y,z),T(z)

1. Resilience

Delete min number of tuples to make Boolean Q false



R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

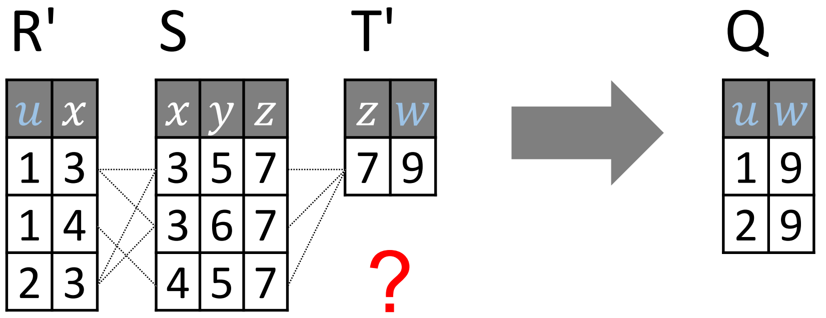
Q

~~true~~

|Γ|=1

5. Smallest witness problem

Delete max num. of tuples while keeping all output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
7	9

?

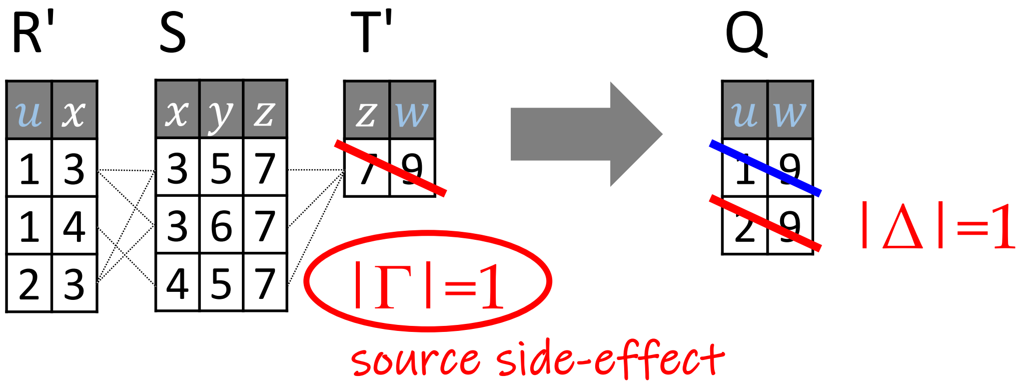
Q

u	w
1	9
2	9

Q(u,w):-R'(u,x),S(x,y,z),T'(z,w)

2/3. (Aggregated) Source side-effects (d.p.)

Delete min number of tuples to delete ≥k output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

Q

u	w
~~1~~	~~9~~
~~2~~	~~9~~

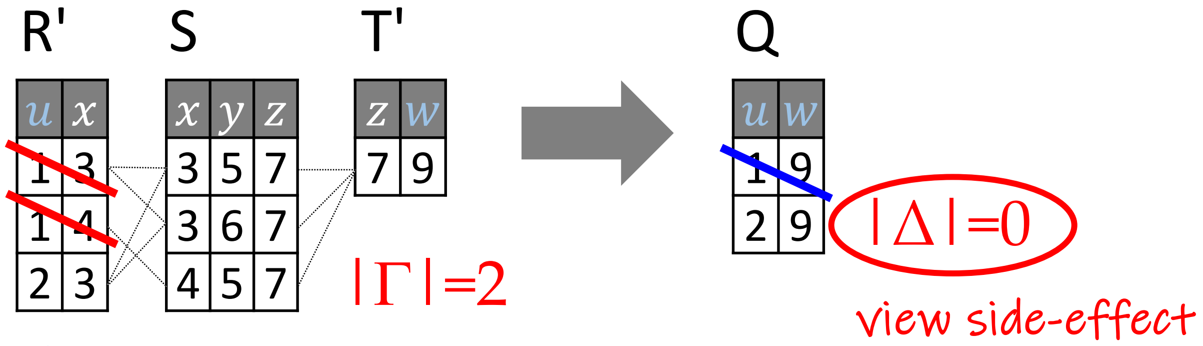
|Δ|=1

|Γ|=1

source side-effect

4/5. (Aggregated) View side-effects (d.p.)

Delete tuples in order to delete ≥k output tuple, while minimizing the other output tuples deleted



R'

u	x
~~1~~	~~3~~
~~1~~	~~4~~
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
7	9

|Γ|=2

Q

u	w
~~1~~	~~9~~
2	9

|Δ|=0

view side-effect

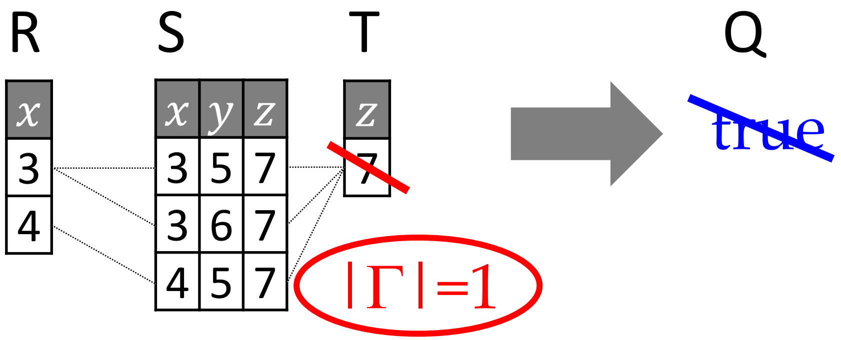
Miao, Roy, Yang. Explaining wrong queries using small examples. SIGMOD, 2019. https://doi.org/10.1145/3299869.3319866
Hu, Sintos. Finding Smallest Witnesses for Conjunctive Queries, ICDT 2024. https://doi.org/10.4230/LIPIcs.ICDT.2024.24

Example Reverse Data Management Problems

Q:-R(x),S(x,y,z),T(z)

1. Resilience

Delete min number of tuples to make Boolean Q false



R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

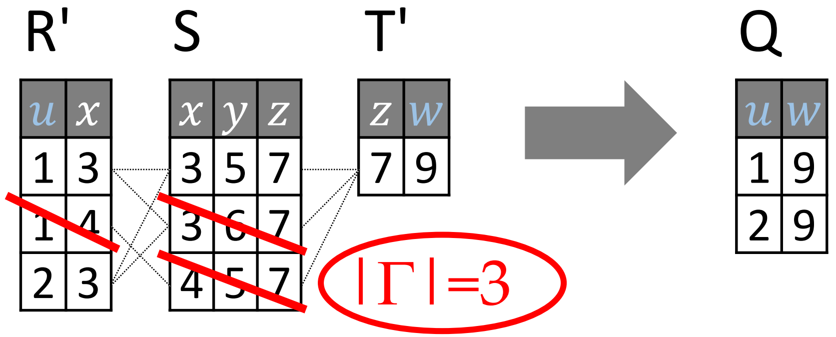
Q

~~true~~

|Γ|=1

5. Smallest witness problem

Delete max num. of tuples while keeping all output tuples



R'

u	x
1	3
~~1~~	~~4~~
2	3

S

x	y	z
3	5	7
~~3~~	~~6~~	~~7~~
~~4~~	~~5~~	~~7~~

T'

z	w
7	9

Q

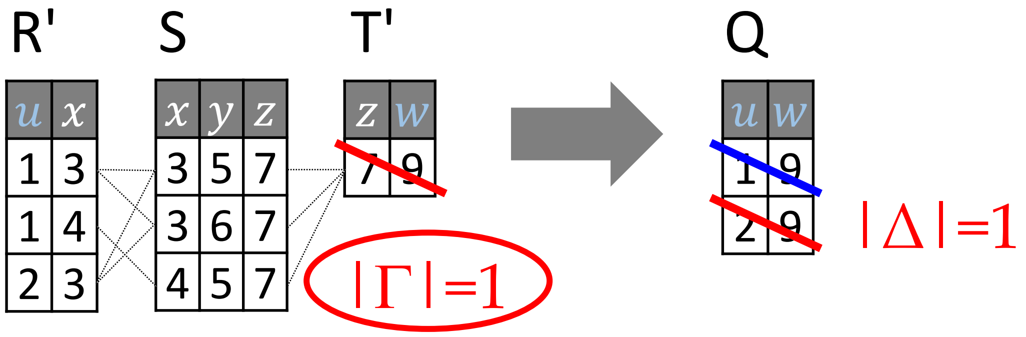
u	w
1	9
2	9

|Γ|=3

Q(u,w):-R'(u,x),S(x,y,z),T'(z,w)

2/3. (Aggregated) Source side-effects (d.p.)

Delete min number of tuples to delete ≥k output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

Q

u	w
~~1~~	~~9~~
~~2~~	~~9~~

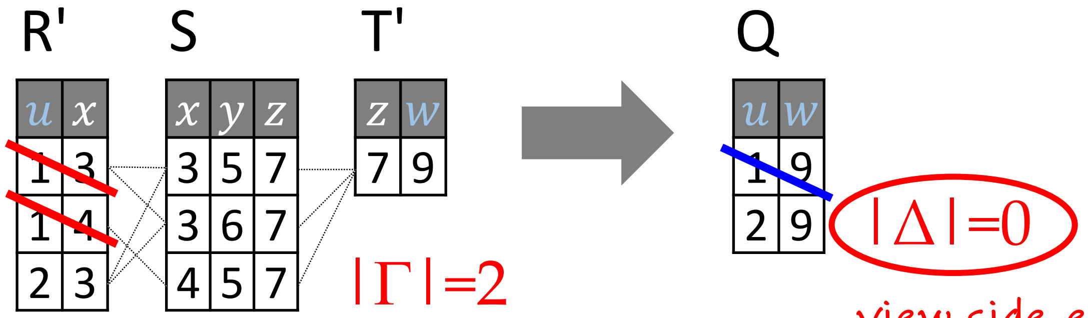
|Γ|=1

|Δ|=1

source side-effect

4/5. (Aggregated) View side-effects (d.p.)

Delete tuples in order to delete ≥k output tuple, while minimizing the other output tuples deleted



R'

u	x
~~1~~	~~3~~
~~1~~	~~4~~
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
7	9

Q

u	w
~~1~~	~~9~~
2	9

|Γ|=2

|Δ|=0

view side-effect

Miao, Roy, Yang. Explaining wrong queries using small examples. SIGMOD, 2019. https://doi.org/10.1145/3299869.3319866

Hu, Sintos. Finding Smallest Witnesses for Conjunctive Queries, ICDT 2024. https://doi.org/10.4230/LIPIcs.ICDT.2024.24

Example Reverse Data Management Problems

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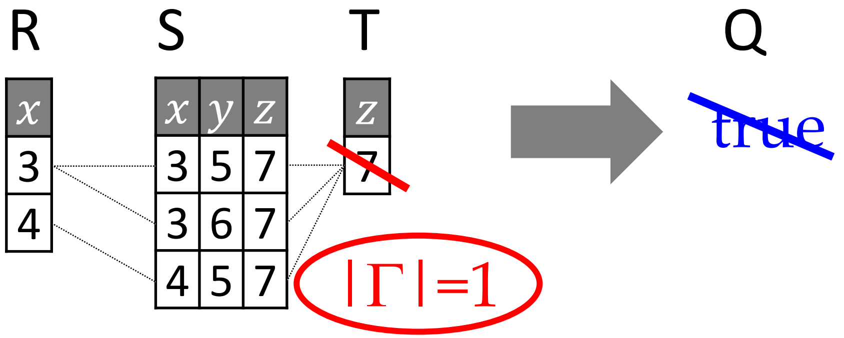


R

x
3
4

S

x	y	z
3	5	7
3	6	7
4	5	7

T

z
~~7~~

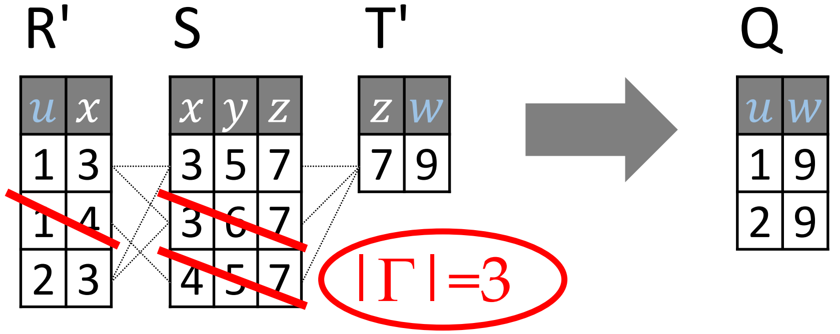
Q

~~true~~

$|\Gamma|=1$

6./7 (Aggregated) Smallest witness problem

Delete max num. of tuples while keeping ≥k output tuples



R'

u	x
1	3
~~1~~	~~4~~
2	3

S

x	y	z
3	5	7
~~3~~	~~6~~	~~7~~
~~4~~	~~5~~	~~7~~

T'

z	w
7	9

Q

u	w
1	9
2	9

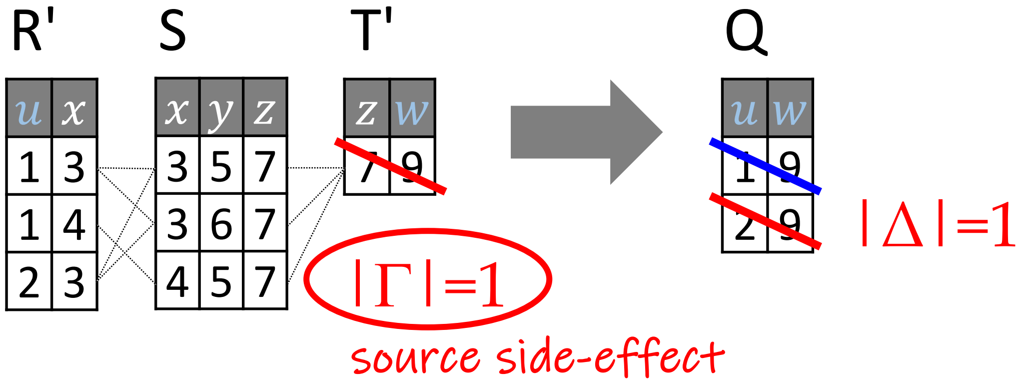
$|\Gamma|=3$

No prior work yet

Q(u,w):-R'(u,x),S(x,y,z),T'(z,w)

2/3. (Aggregated) Source side-effects (d.p.)

Delete min number of tuples to delete ≥k output tuples



R'

u	x
1	3
1	4
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
~~7~~	~~9~~

Q

u	w
~~1~~	~~9~~
~~2~~	~~9~~

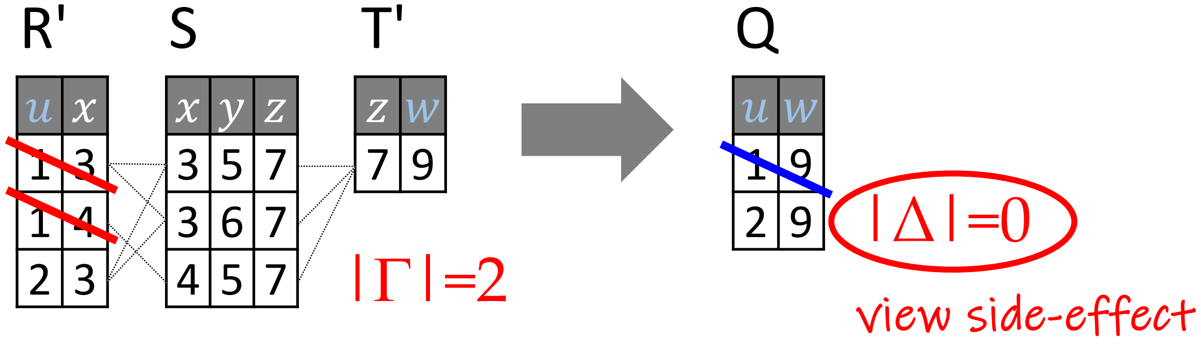
$|\Delta|=1$

$|\Gamma|=1$

source side-effect

4/5. (Aggregated) View side-effects (d.p.)

Delete tuples in order to delete ≥k output tuple, while minimizing the other output tuples deleted



R'

u	x
~~1~~	~~3~~
~~1~~	~~4~~
2	3

S

x	y	z
3	5	7
3	6	7
4	5	7

T'

z	w
7	9

Q

u	w
~~1~~	~~9~~
2	9

$|\Gamma|=2$

$|\Delta|=0$

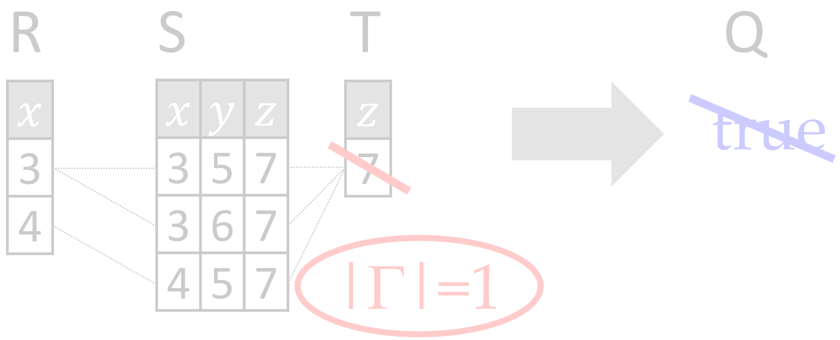
view side-effect

A Plethora of Reverse Data Management Problems ...

Q:-R(x),S(x,y,z),T(z)

1. Resilience

Delete min number of tuples to make Boolean Q false



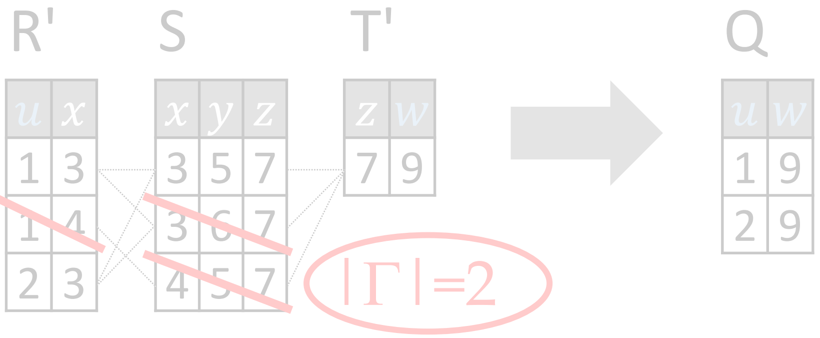
x
3
4

x	y	z
3	5	7
3	6	7
4	5	7

z
~~7~~

6./7 (Aggregated) Smallest witness problem

Delete max num. of tuples while keeping ≥k output tuples



u	x
1	3
~~1~~	~~4~~
2	3

x	y	z
3	5	7
~~3~~	~~6~~	~~7~~
~~4~~	~~5~~	~~7~~

z	w
7	9

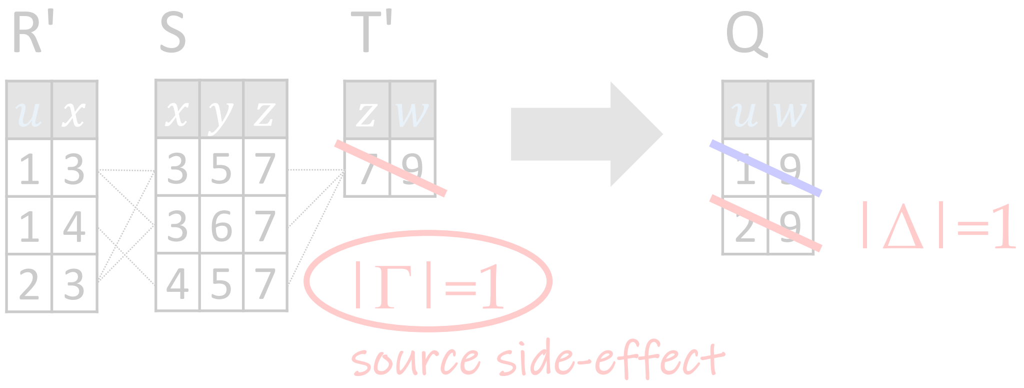
u	w
1	9
2	9

?

Q(u,w):-R'(u,x),S(x,y,z),T'(z,w)

2/3. (Aggregated) Source side-effects (d.p.)

Delete min number of tuples to delete ≥k output tuples



u	x
1	3
1	4
2	3

x	y	z
3	5	7
3	6	7
4	5	7

z	w
~~7~~	~~9~~

u	w
~~1~~	~~9~~
~~2~~	~~9~~

4/5. (Aggregated) View side-effects (d.p.)

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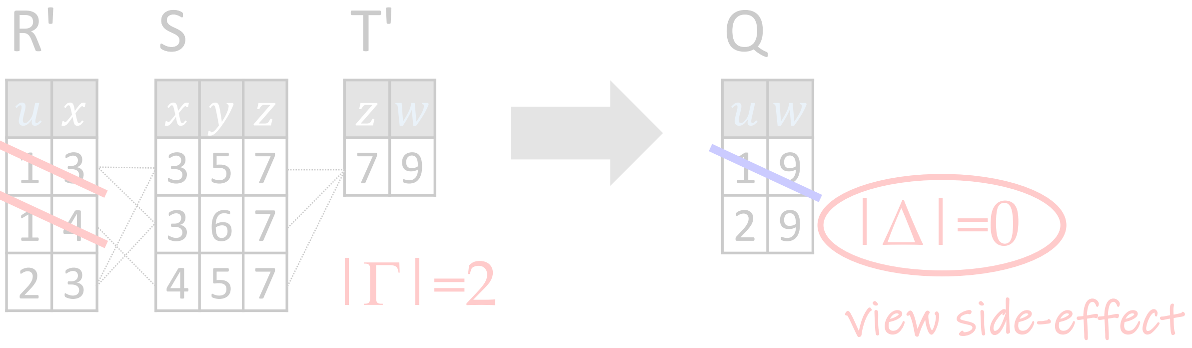


u	x
~~1~~	~~3~~
~~1~~	~~4~~
2	3

x	y	z
3	5	7
3	6	7
4	5	7

z	w
7	9

u	w
~~1~~	~~9~~
2	9

Our topic today: Generalized Deletion Propagation

1. Unify and generalize these problems as instances of "generalized deletion propagation"
    - also allows new problem variants

2. Propose one **ILP encoding** to solve all these problems
    - including difficult cases, such as self-joins, or bag semantics

3. The ILP formulation **is solvable in PTIME for all known PTIME cases**
    - including all known PTIME cases for the problems of: **resilience** [VLDB'15], **aggregated deletion propagation** [VLDB'20], **view-side effects** [PODS'11], **smallest witness problem** [ICDT'24], under functional dependencies, and both set and bag semantics (where results are known)

Outline

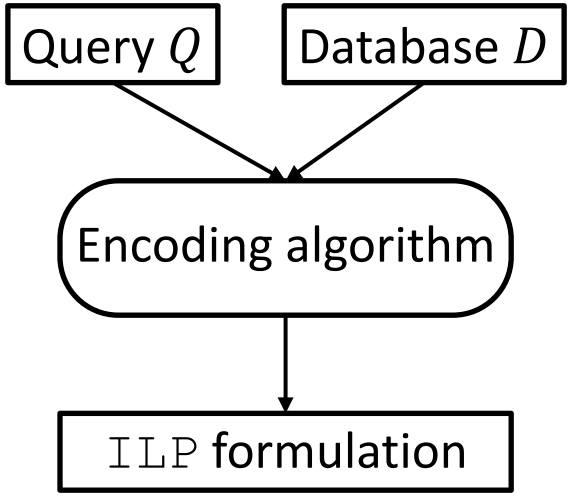
1. Reverse Data Management (RDM)
2. A magical ILP formulation
3. Take-aways

(An illustrated example: only if time remains)

Unified Algorithms for Reverse Data Management

`ILP` formulation:

$$
\begin{aligned}
f^* = \min[\mathbf{c} \cdot \mathbf{x}] \\
\text{s.t. } \mathbf{A} \cdot \mathbf{x} \geq \mathbf{b} \\
\mathbf{x} \in \mathbb{N}^n
\end{aligned}
$$



Unified Algorithms for Reverse Data Management

`ILP` formulation:

$$f^* = \min[\mathbf{c} \cdot \mathbf{x}]$$
$$\text{s.t. } \mathbf{A} \cdot \mathbf{x} \geq \mathbf{b}$$
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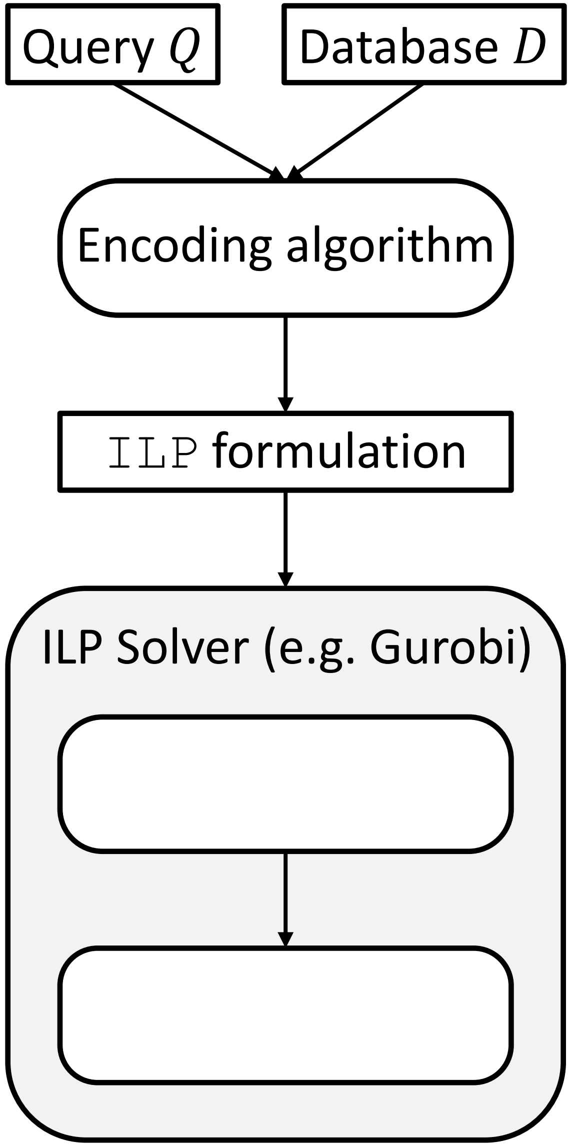

Query Q

Database D

Encoding algorithm

`ILP` formulation

ILP Solver (e.g. Gurobi)

Unified Algorithms for Reverse Data Management

`ILP` formulation:

$$f^* = \min[\mathbf{c} \cdot \mathbf{x}]$$

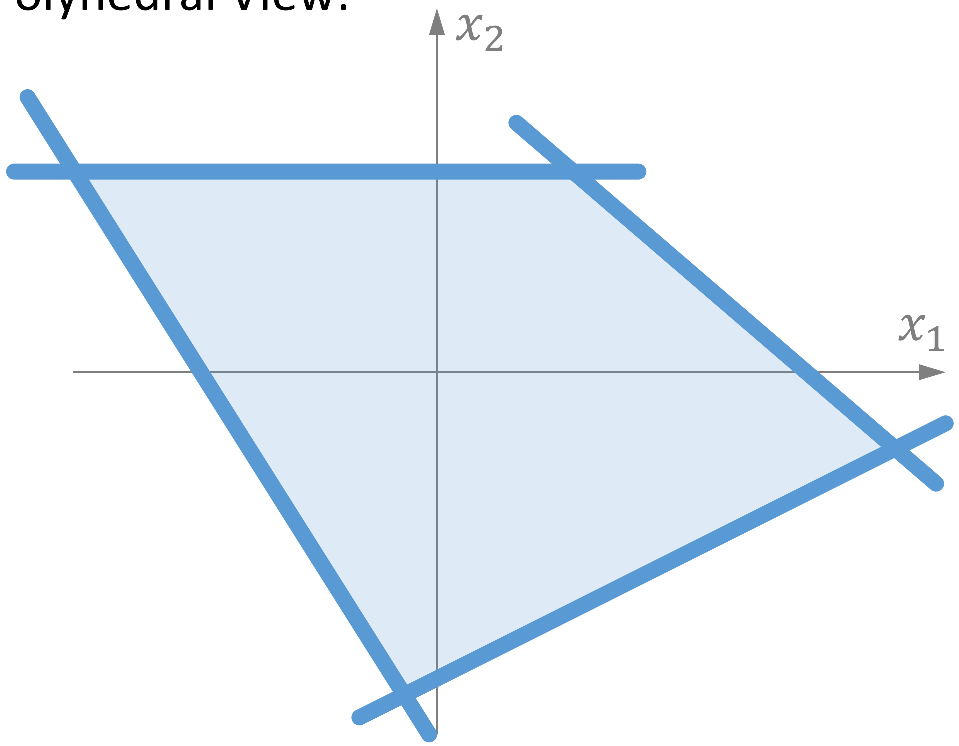
constraint vector

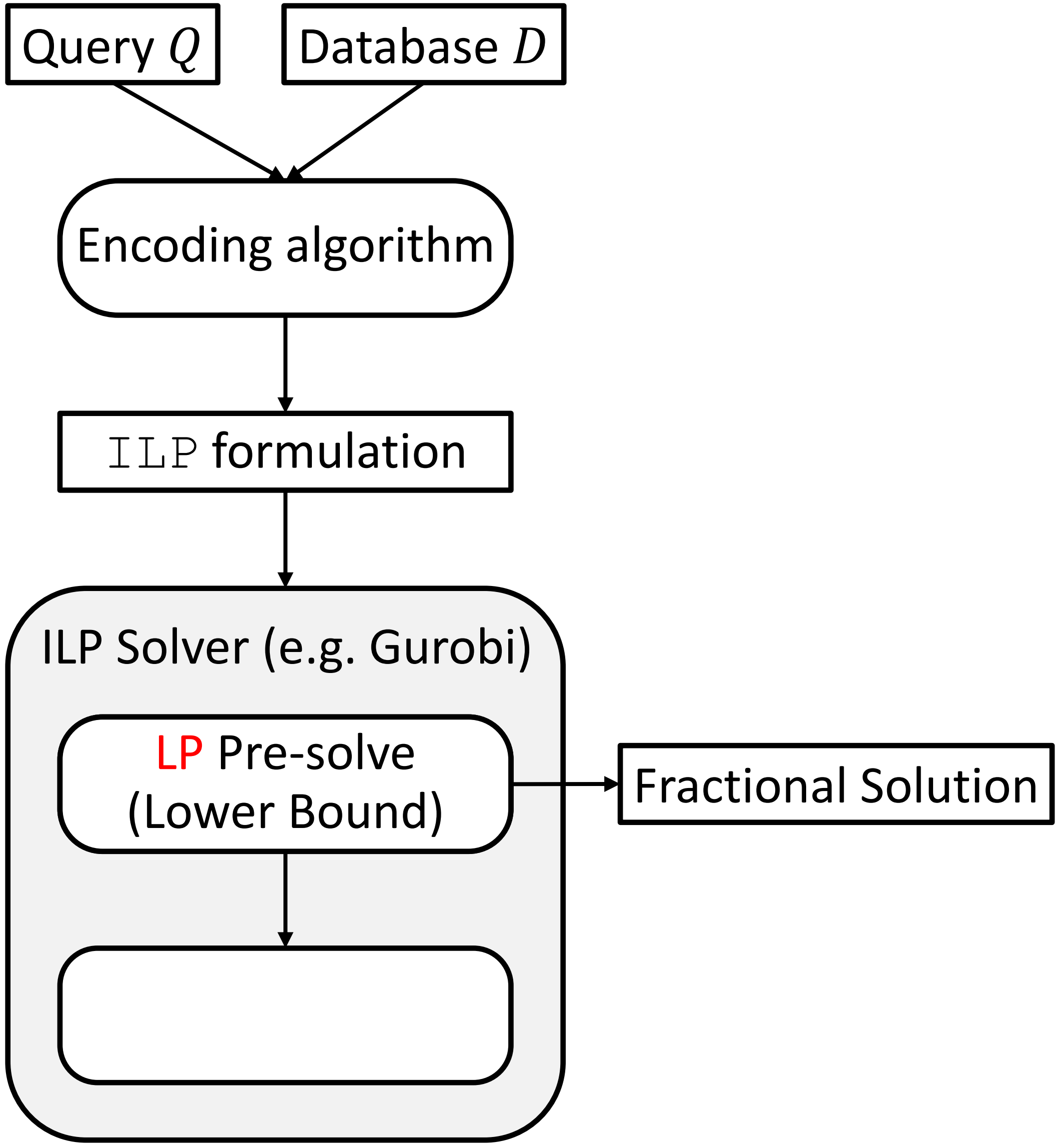
$$\text{s.t. } \mathbf{A} \cdot \mathbf{x} \geq \mathbf{b}$$

constraint matrix

$$\mathbf{x} \in \mathbb{N}^n / \mathbb{R}^n$$

Polyhedral View:





Query Q

Database D

Encoding algorithm

`ILP` formulation

ILP Solver (e.g. Gurobi)

LP Pre-solve
(Lower Bound)

Fractional Solution

Unified Algorithms for Reverse Data Management

`ILP` formulation:

objective vector

$$f^* = \min[\mathbf{c} \cdot \mathbf{x}]$$

constraint vector

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constraint matrix

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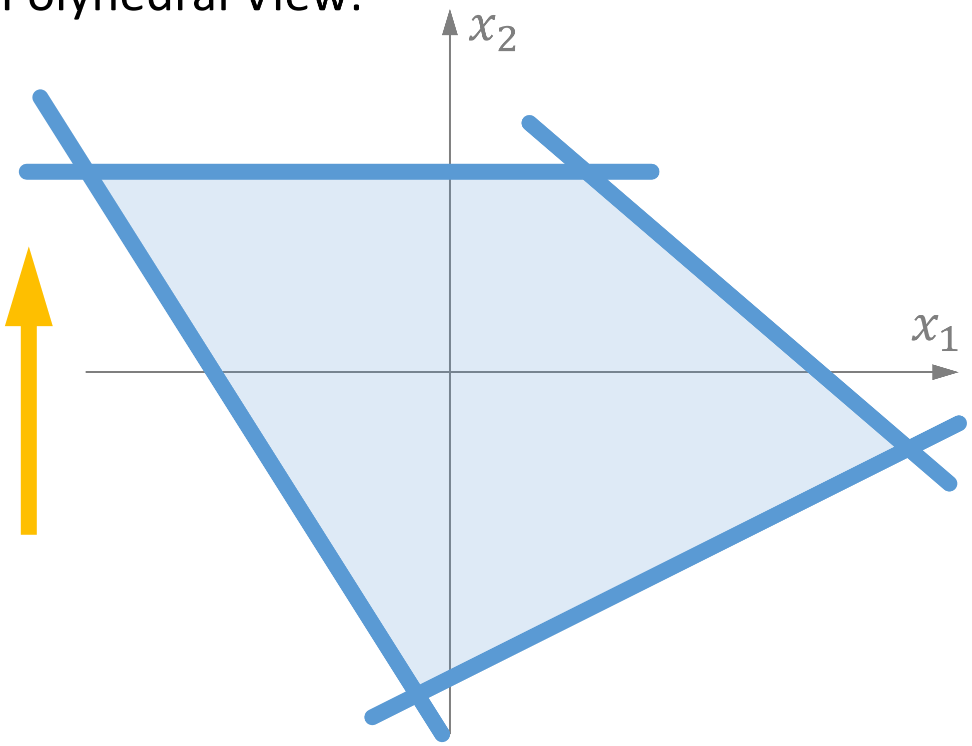
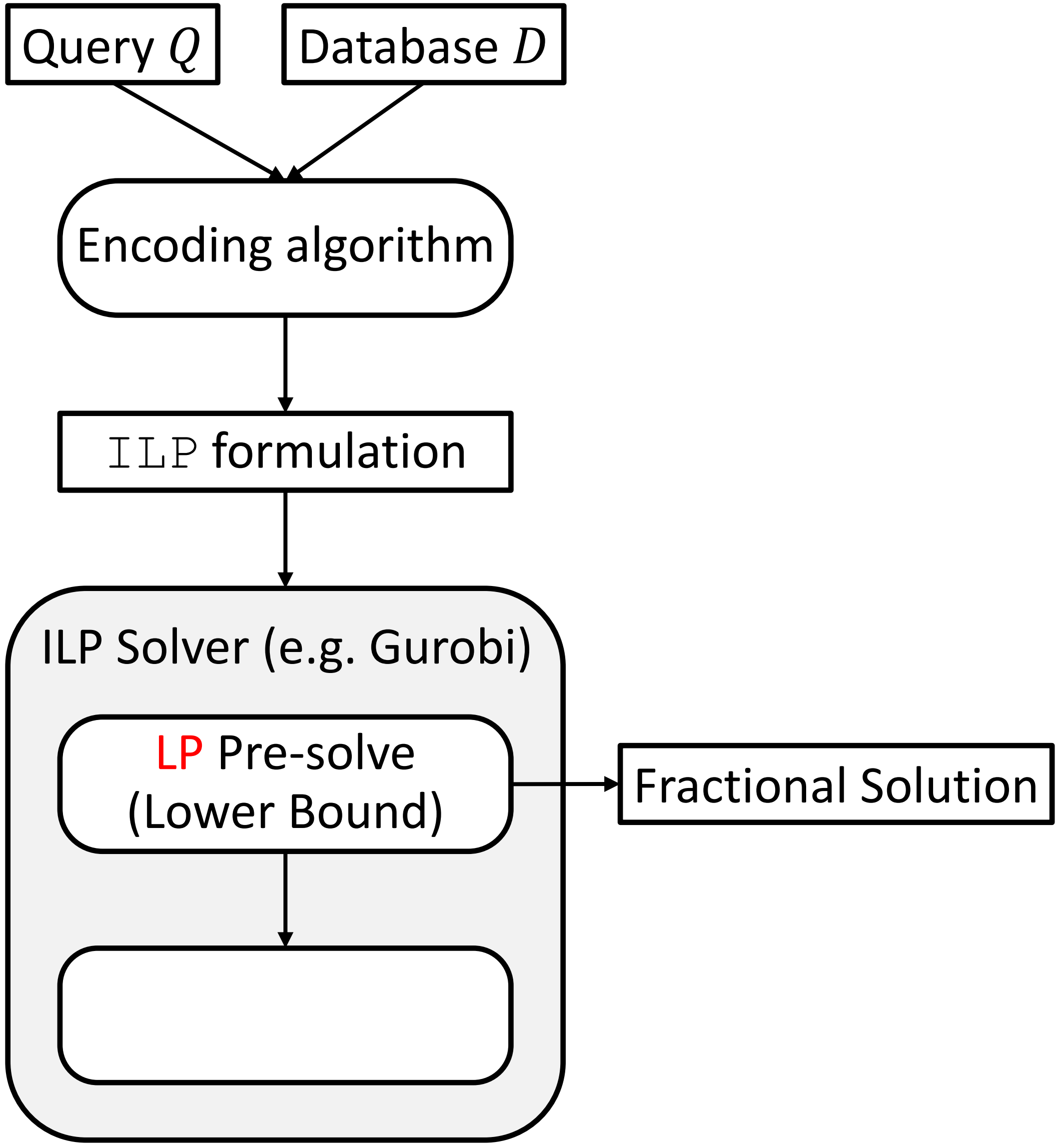
Polyhedral View:





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Fractional Solution

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objective vector

constraint vector

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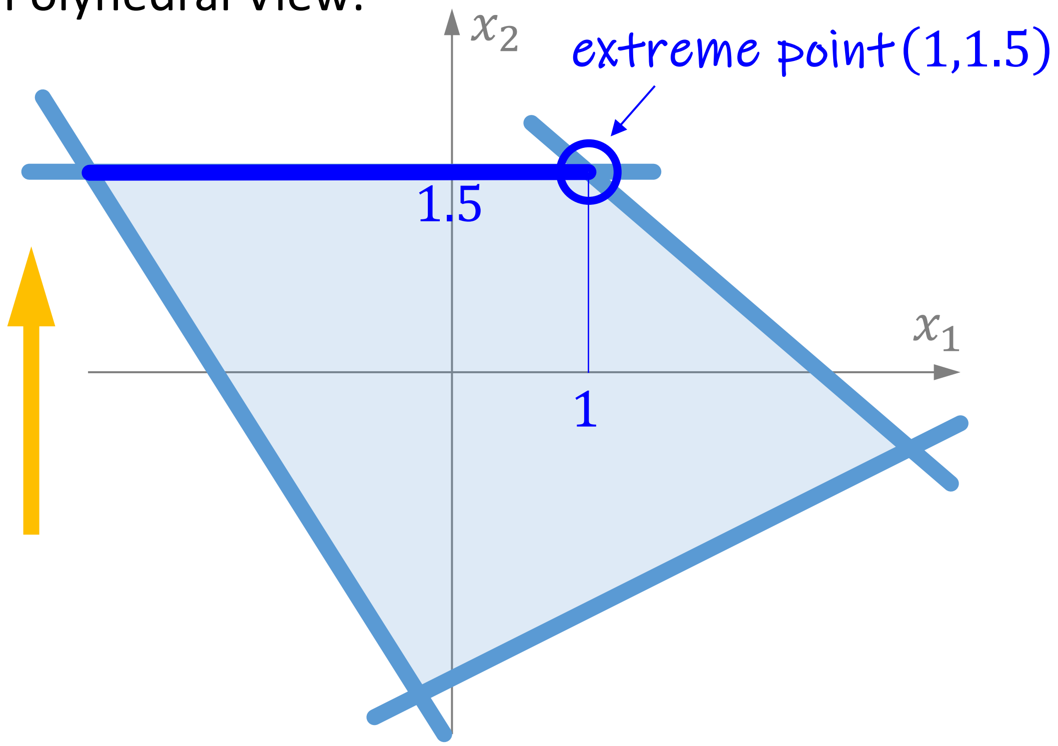
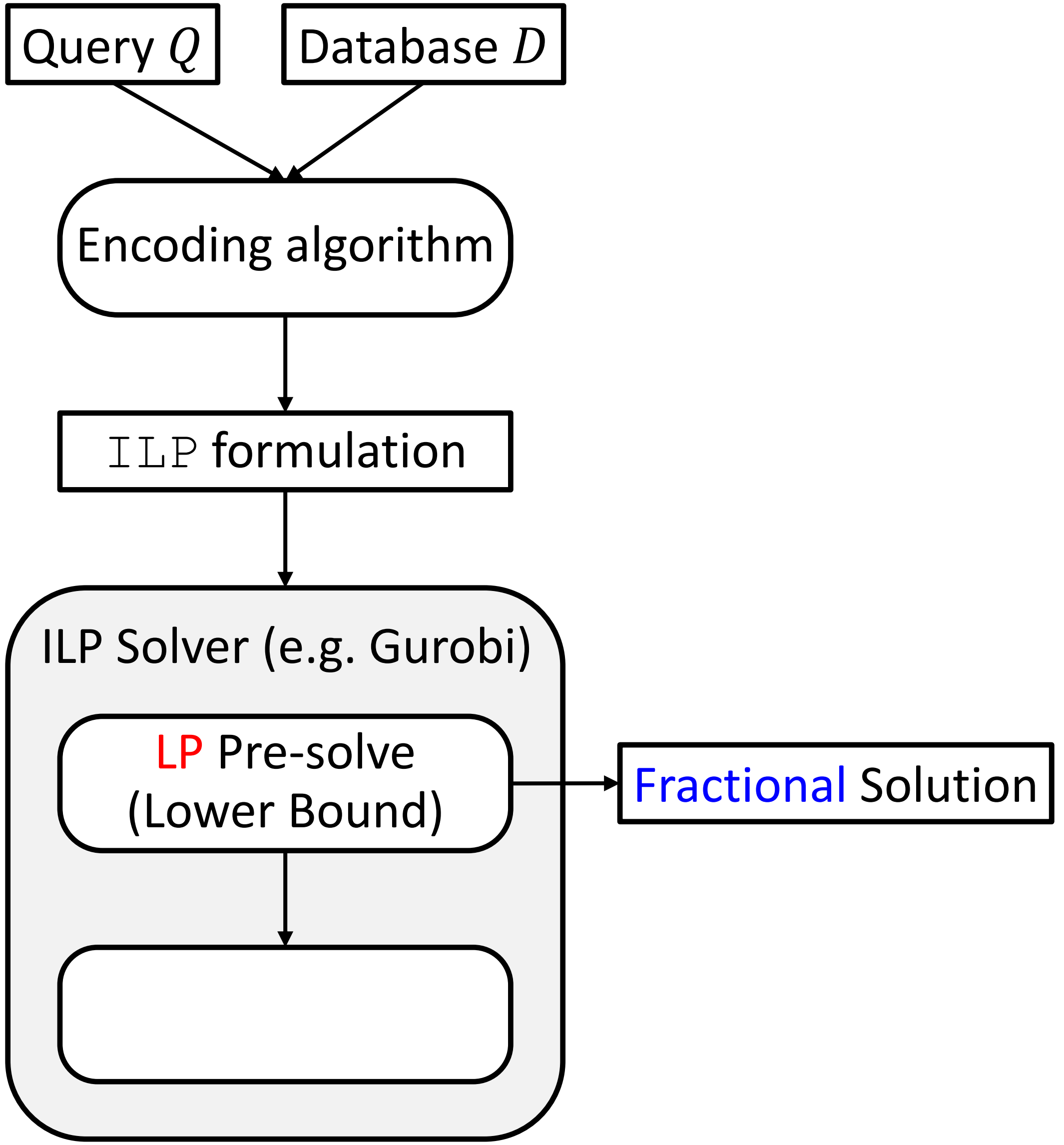
$$\mathbf{x} \in \mathbb{N}^n / \mathbb{R}^n$$

constraint matrix

Polyhedral View:




Query Q

Database D

Encoding algorithm

`ILP` formulation

ILP Solver (e.g. Gurobi)

LP Pre-solve
(Lower Bound)

Fractional Solution

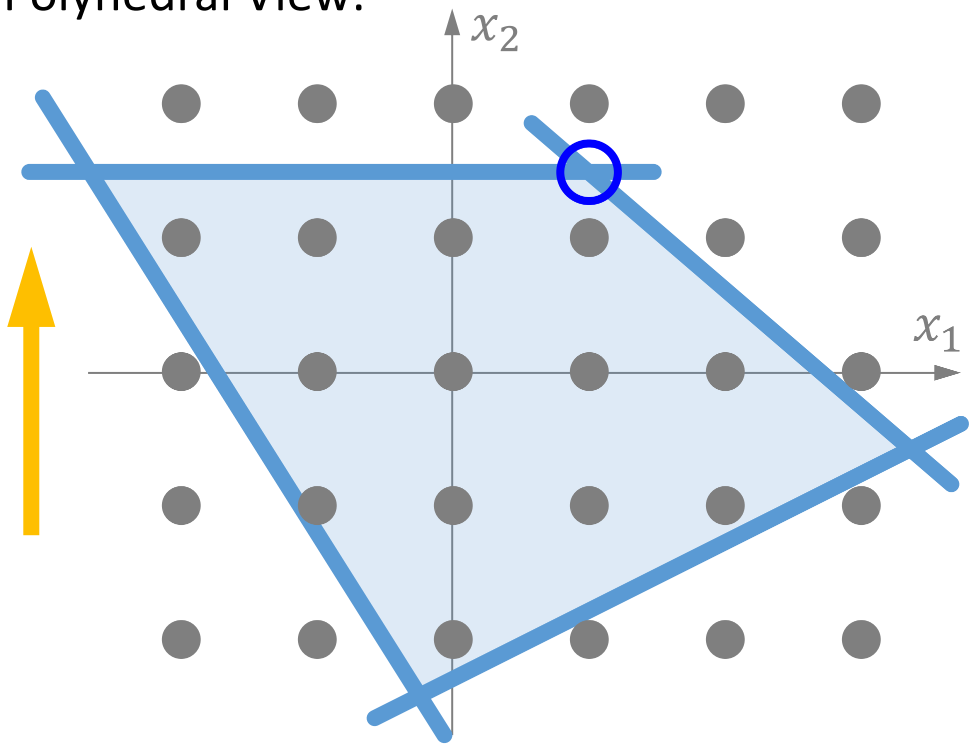
f^*=1.5
e.g. $\mathbf{x}^*$=(1,1.5)

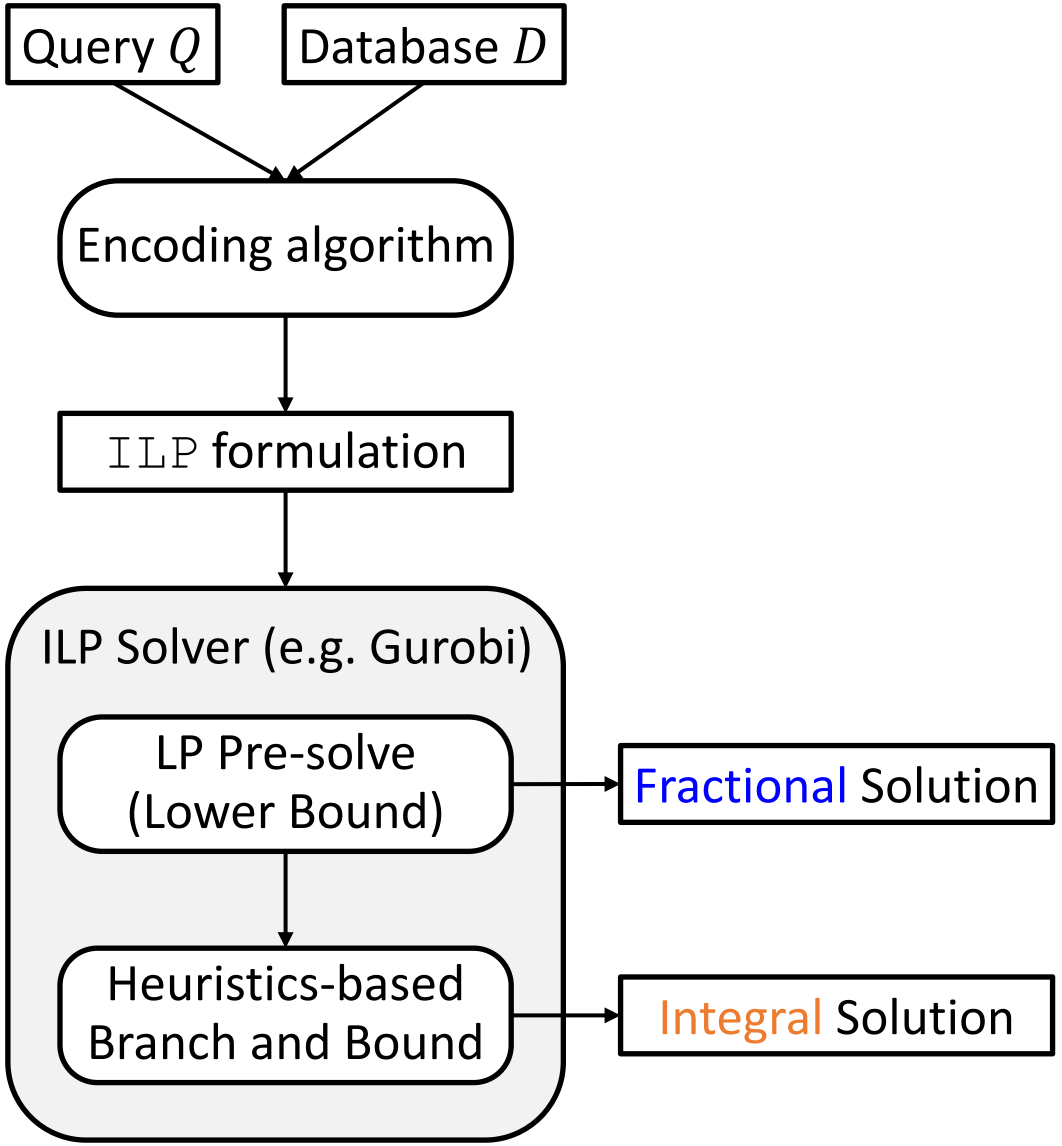
Unified Algorithms for Reverse Data Management

`ILP` formulation:

$$\begin{aligned} f^* &= \min[\mathbf{c}\cdot\mathbf{x}] \\ \text{s.t. } & \mathbf{A}\cdot\mathbf{x} \geq \mathbf{b} \\ & \mathbf{x} \in \mathbb{N}^n/\mathbb{R}^n \end{aligned}$$

Polyhedral View:





Query Q

Database D

Encoding algorithm

`ILP` formulation

ILP Solver (e.g. Gurobi)

LP Pre-solve
(Lower Bound)

Heuristics-based
Branch and Bound

Fractional Solution

Integral Solution

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e.g. $\mathbf{x}^*$=(1,1.5)

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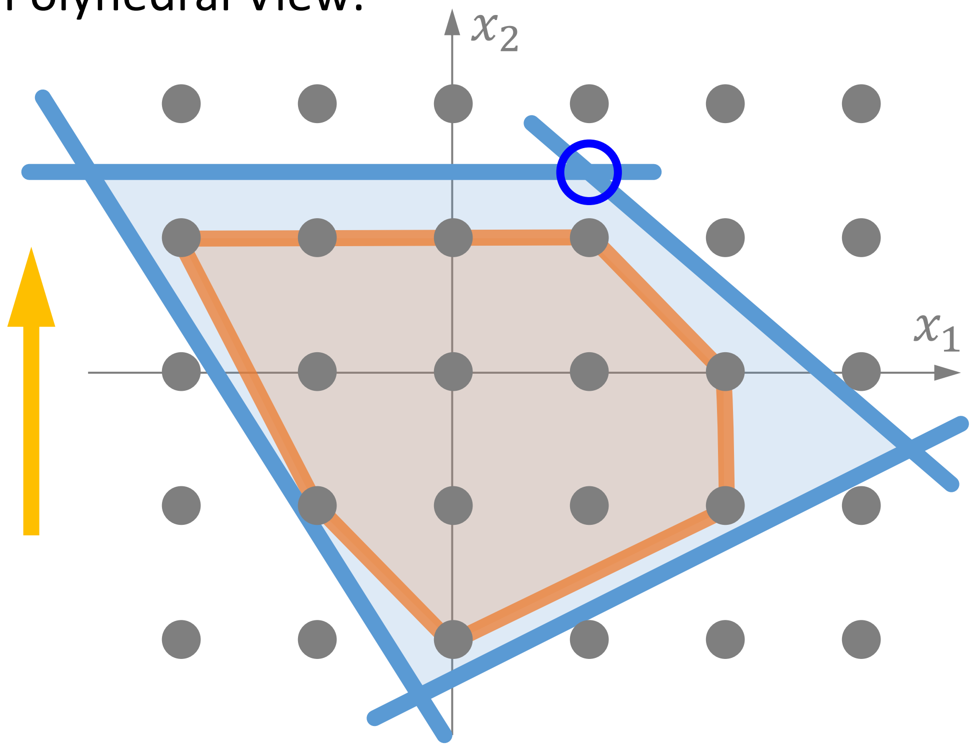

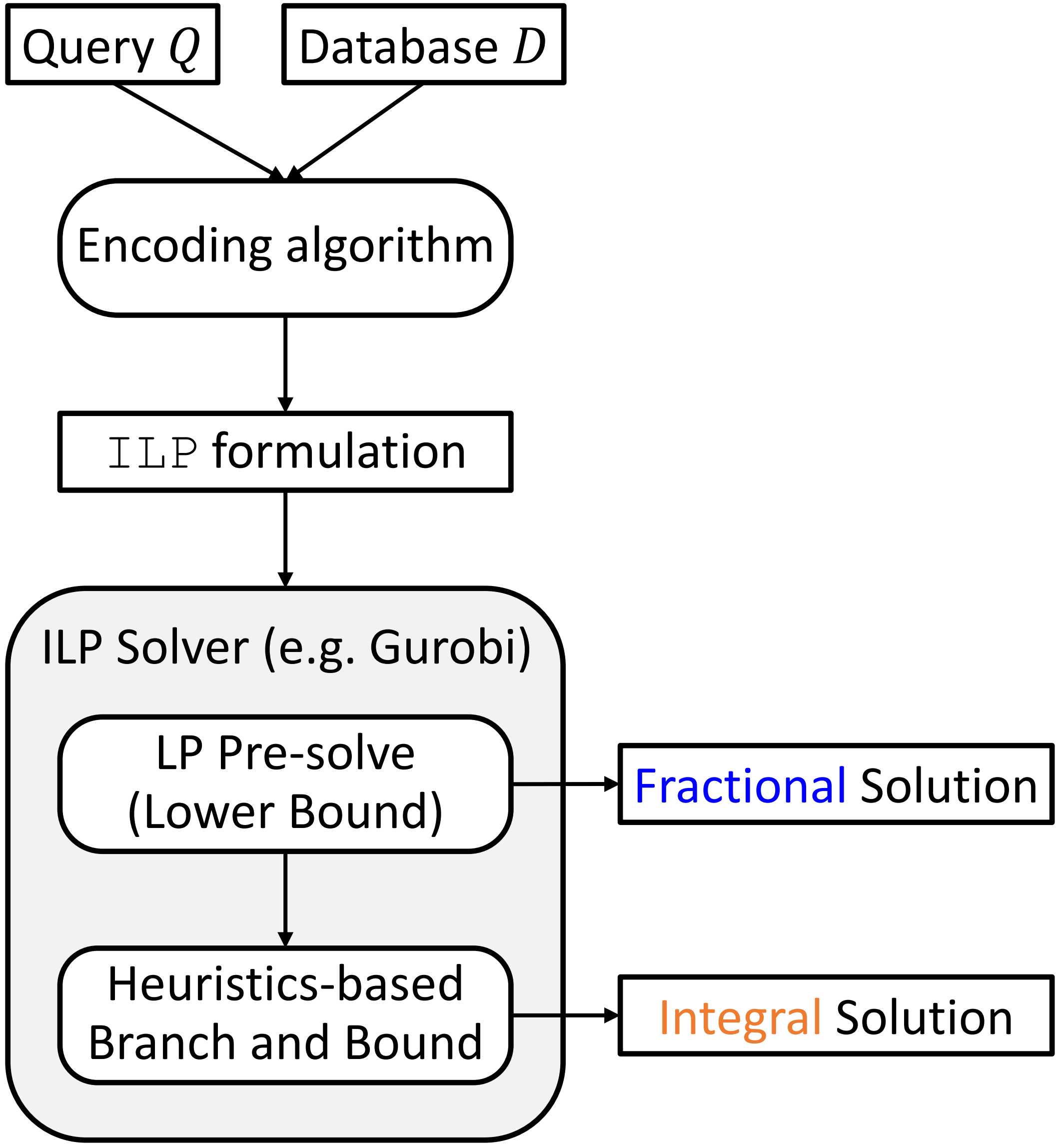

Query Q

Database D

Encoding algorithm

`ILP` formulation

ILP Solver (e.g. Gurobi)

LP Pre-solve (Lower Bound)

Heuristics-based Branch and Bound

Fractional Solution

Integral Solution

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e.g. $\mathbf{x}^*$=(1,1.5)

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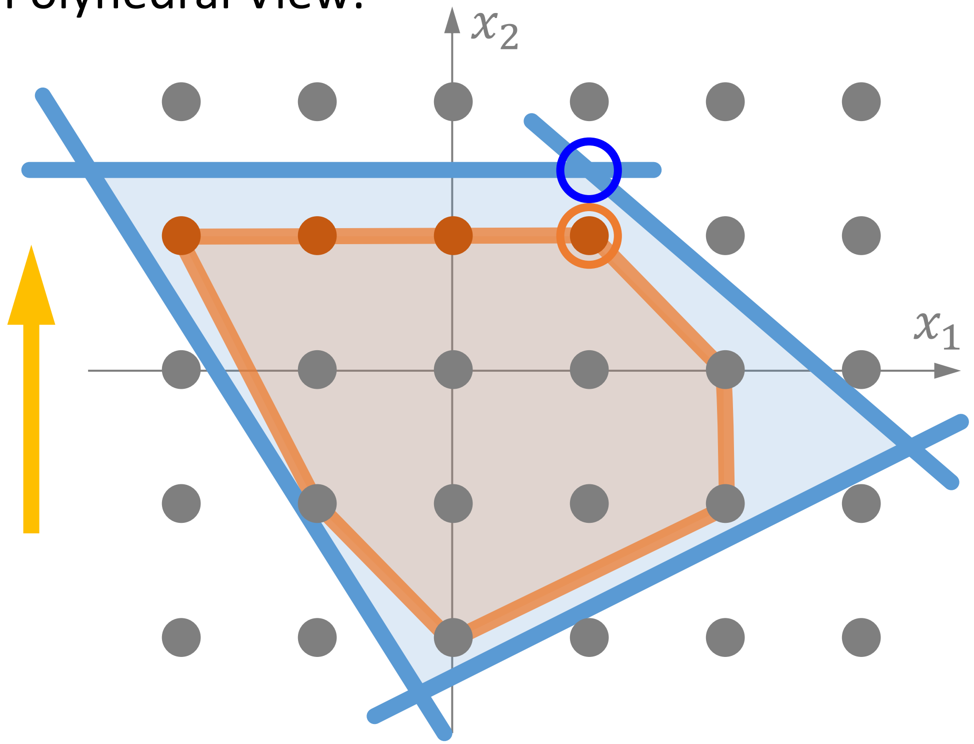

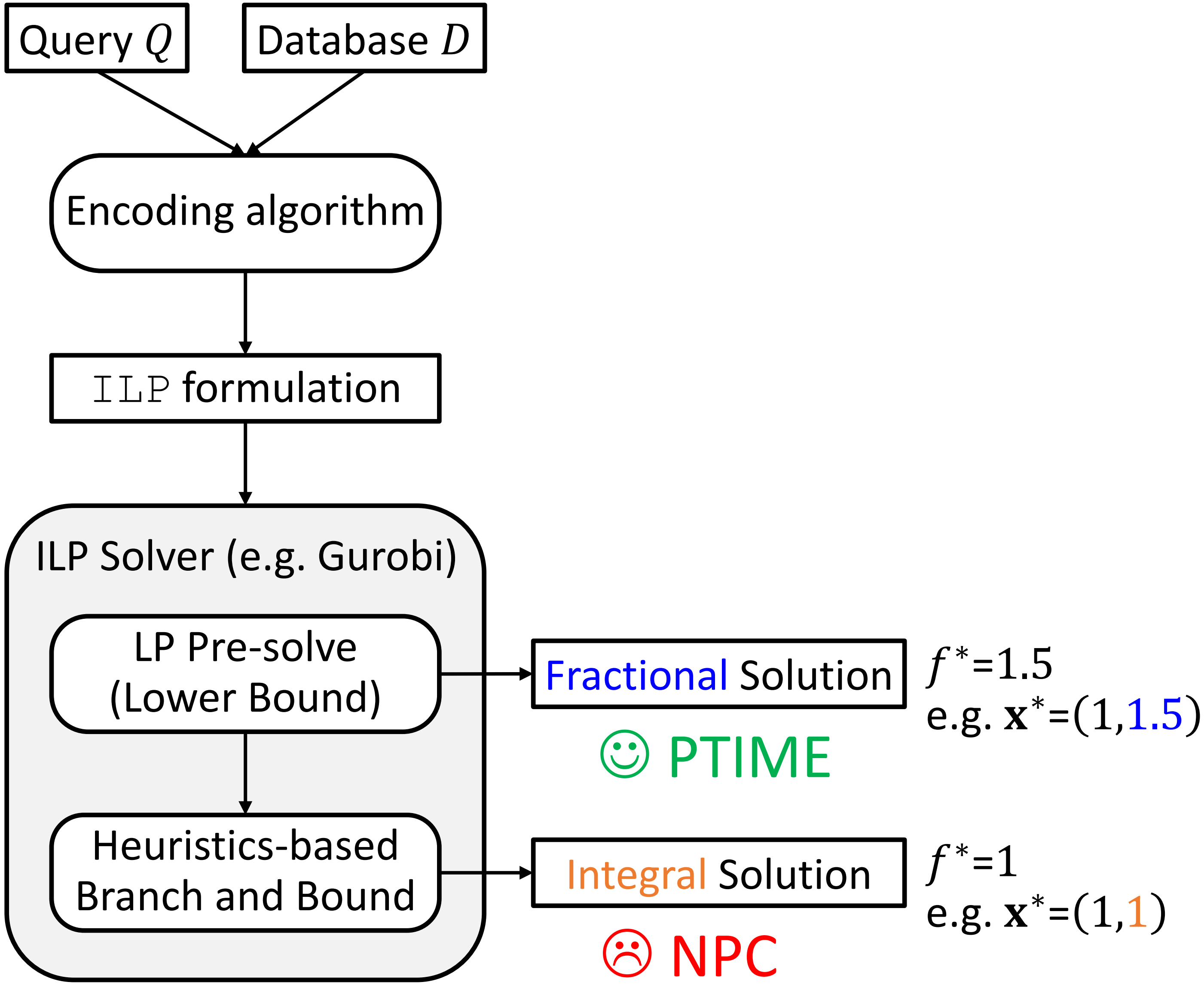

Query Q

Database D

Encoding algorithm

`ILP` formulation

ILP Solver (e.g. Gurobi)

LP Pre-solve (Lower Bound)

Heuristics-based Branch and Bound

Fractional Solution

f^*=1.5

e.g. $\mathbf{x}^*$=(1,1.5)

☺ PTIME

Integral Solution

f^*=1

e.g. $\mathbf{x}^*$=(1,1)

☹ NPC

Unified Algorithms for Reverse Data Management

`ILP` formulation:

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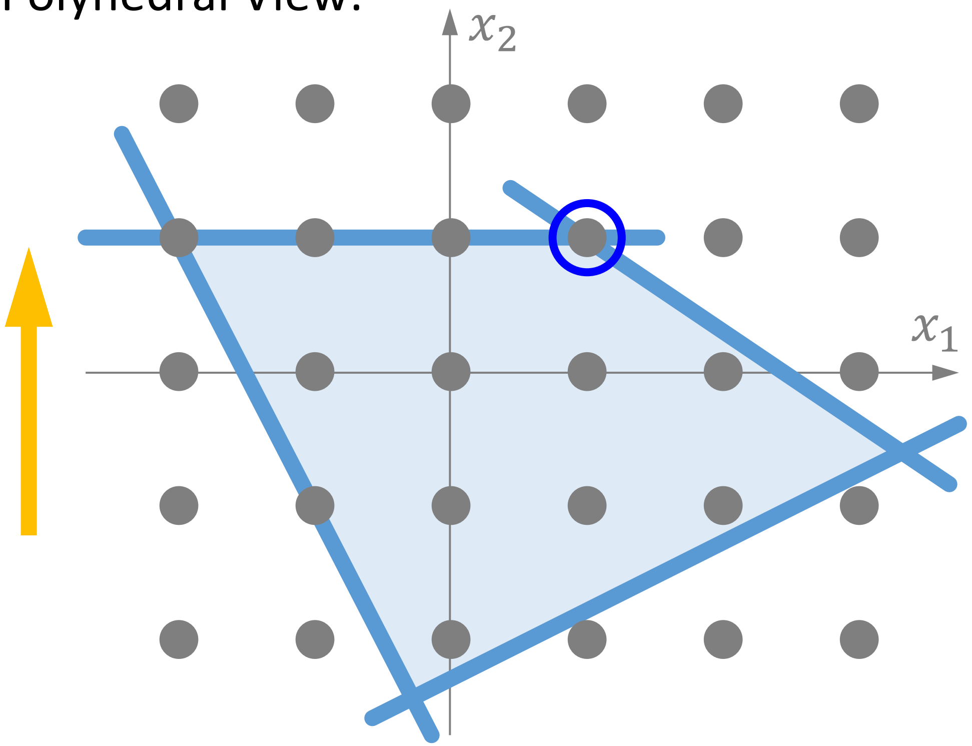
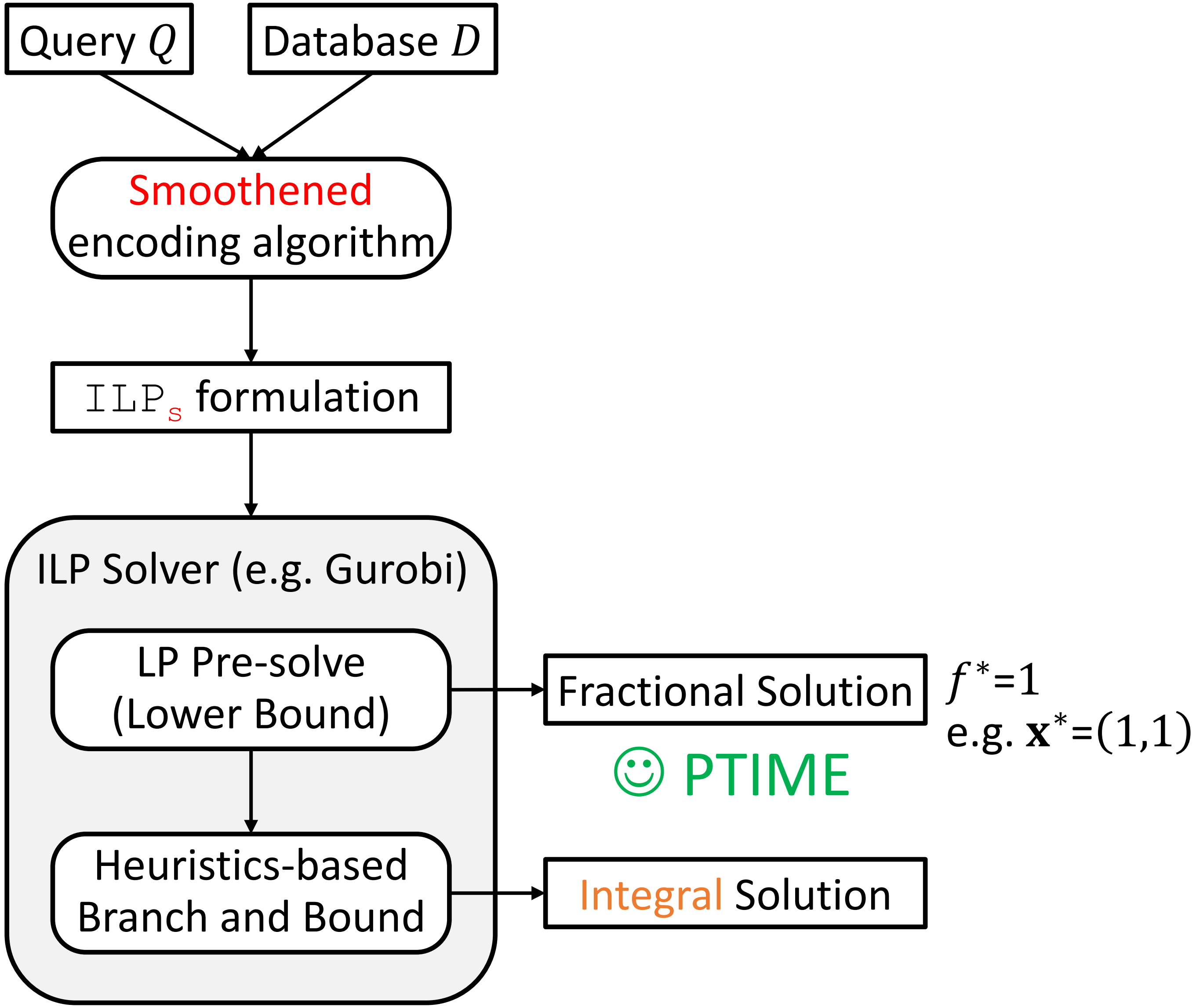
Polyhedral View:





Query Q

Database D

Smoothened encoding algorithm

$\texttt{ILP}_s$ formulation

ILP Solver (e.g. Gurobi)

LP Pre-solve (Lower Bound)

Heuristics-based Branch and Bound

Fractional Solution

f^*=1
e.g. $\mathbf{x}^*$=(1,1)

☺ PTIME

Integral Solution

Unified Algorithms for Reverse Data Management

`ILP` formulation:

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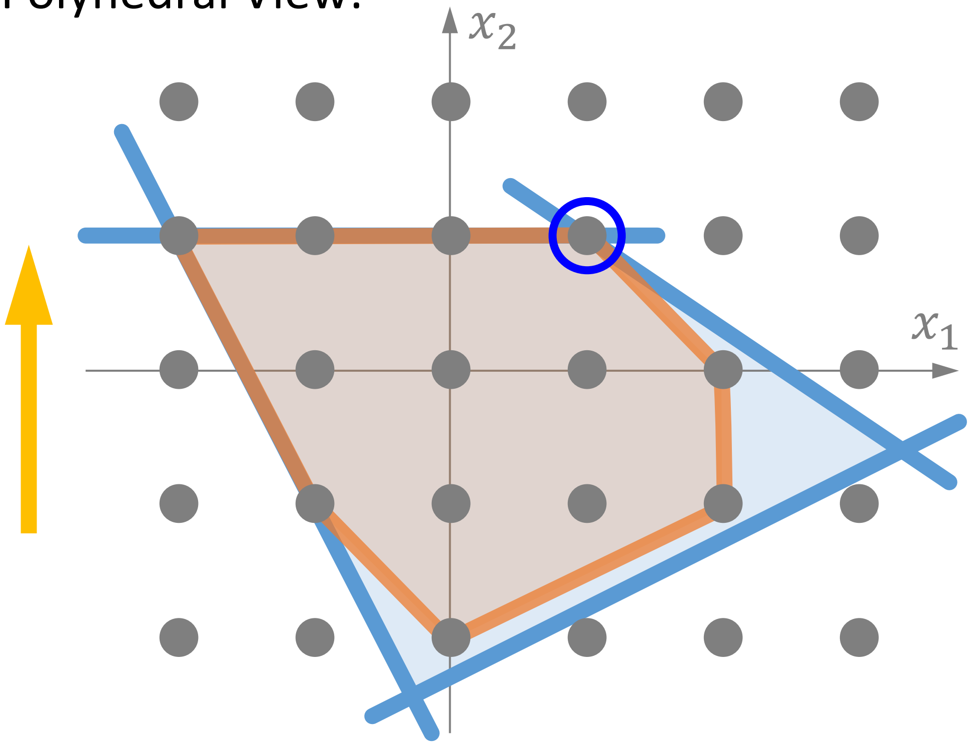
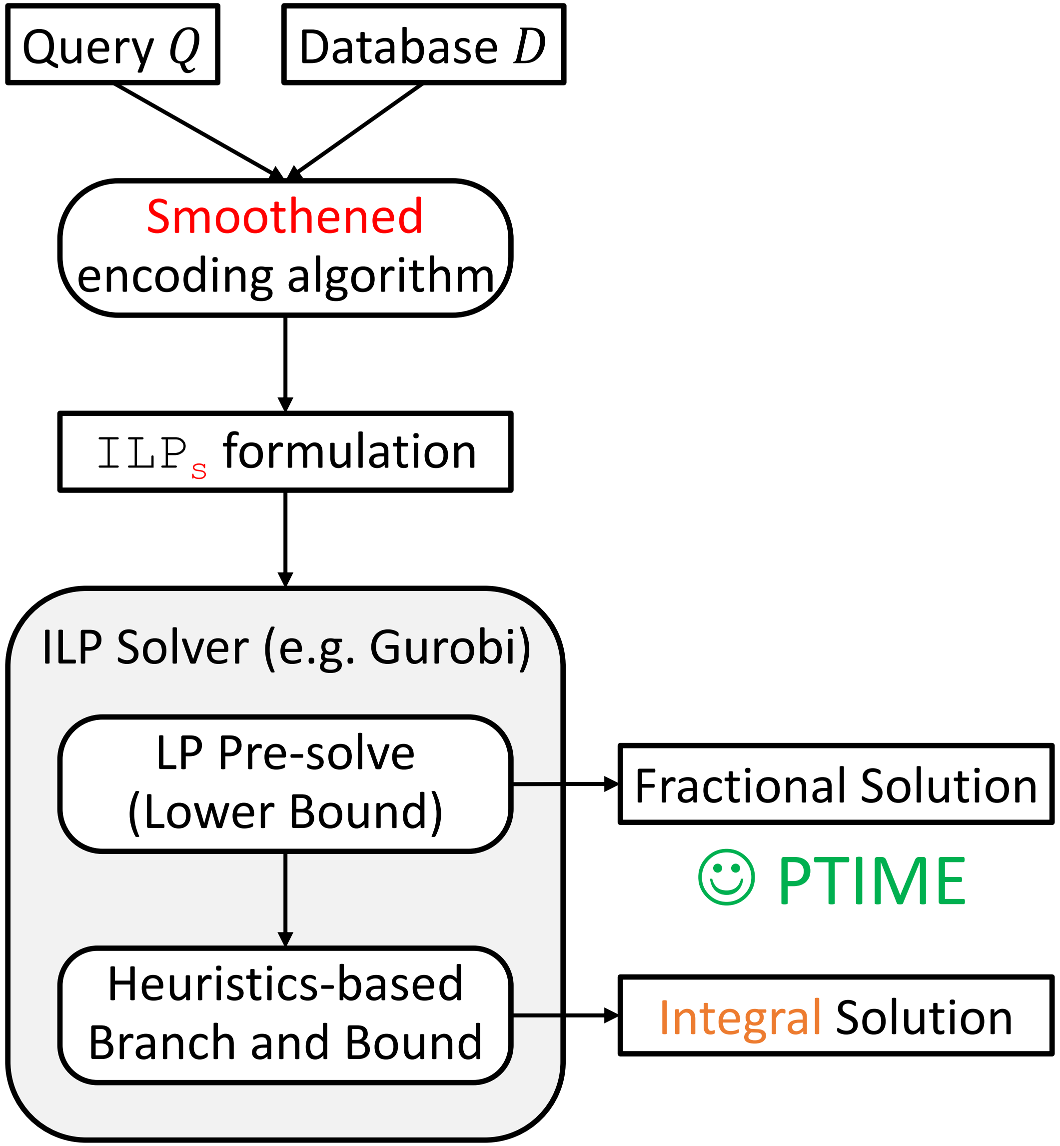
$$\text{s.t. } \mathbf{A} \cdot \mathbf{x} \geq \mathbf{b}$$
$$\mathbf{x} \in \mathbb{N}^n / \mathbb{R}^n$$

Polyhedral View:





f^*=1
e.g. $\mathbf{x}^*$=(1,1)

☺ PTIME

Unified Algorithms for Reverse Data Management

`ILP` formulation:

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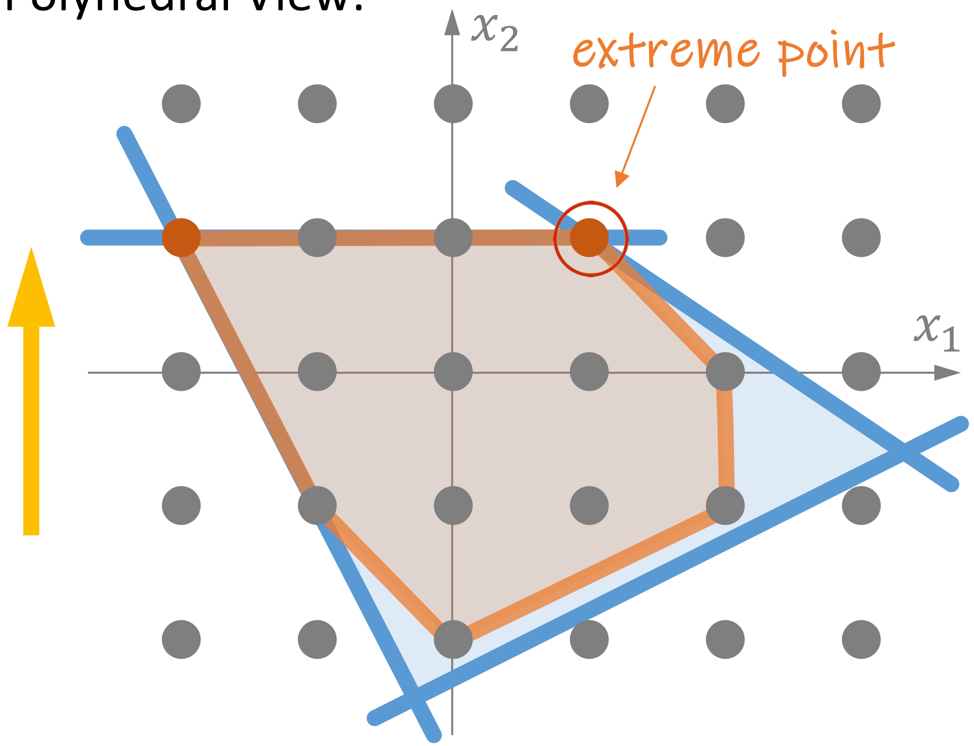
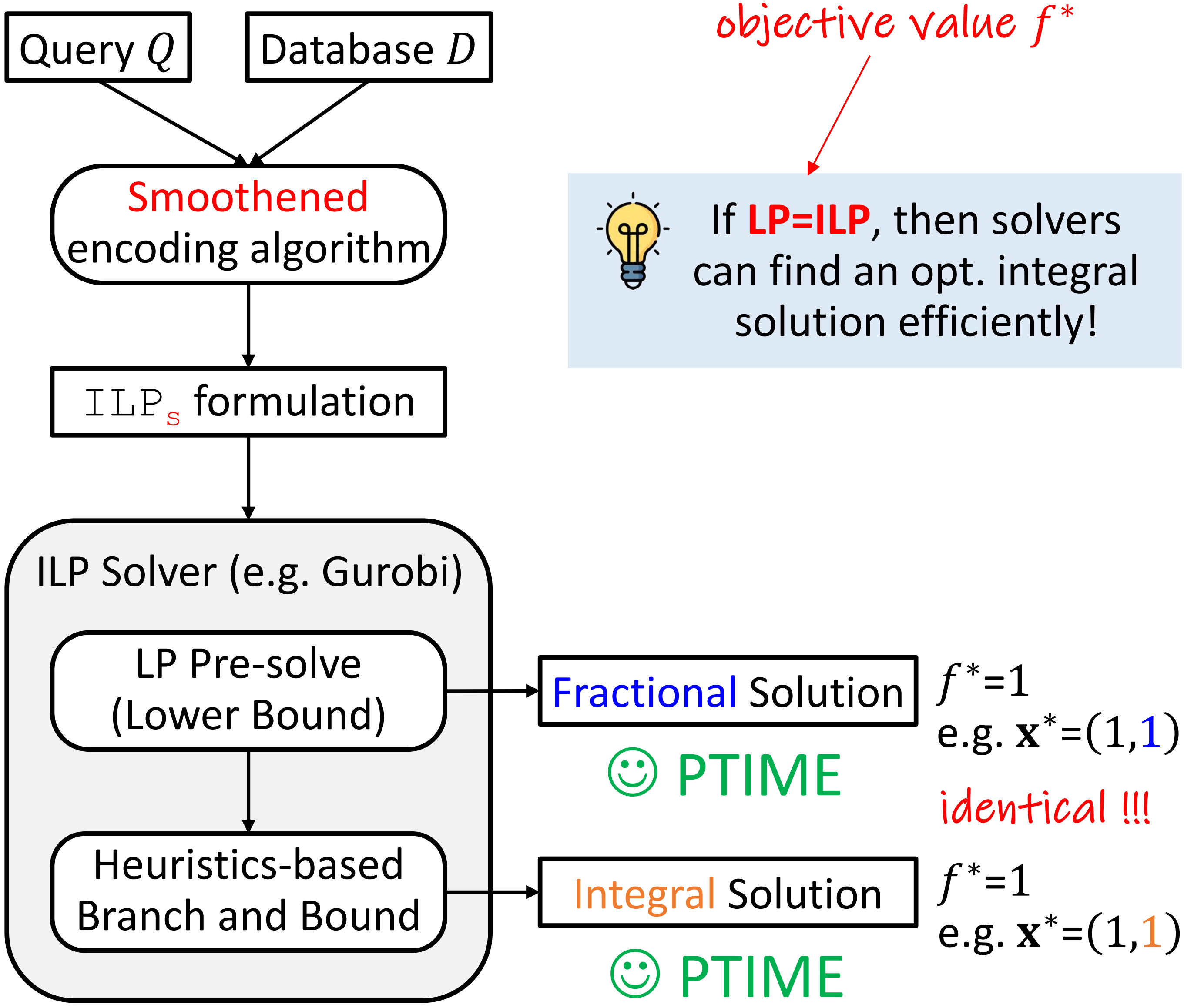
$$\mathbf{x} \in \mathbb{N}^n / \mathbb{R}^n$$

Polyhedral View:



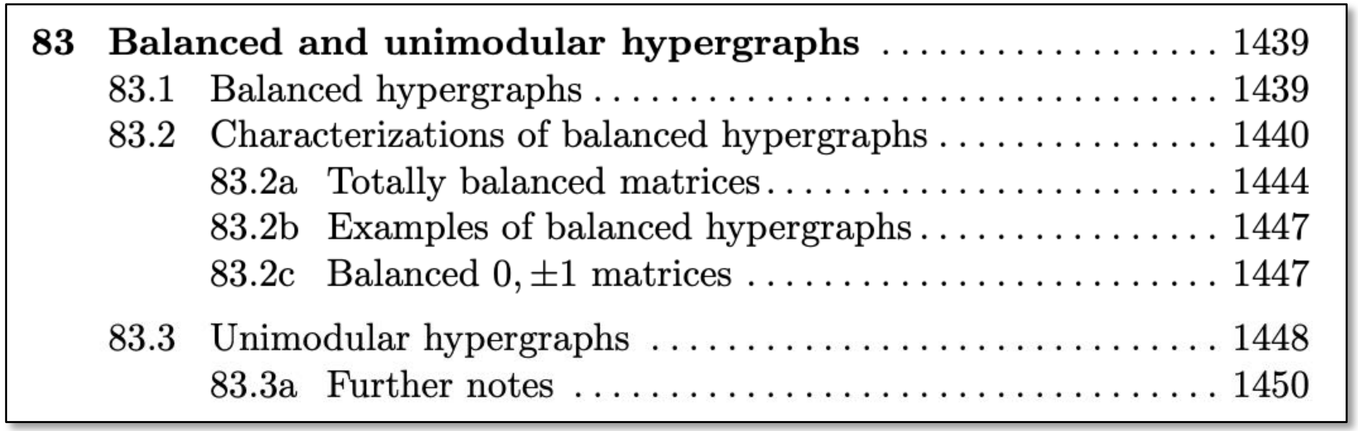


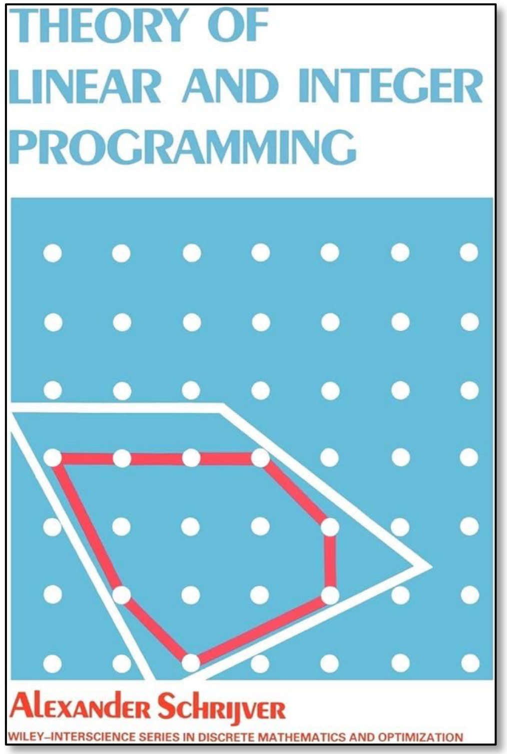
If **LP=ILP**, then solvers can find an opt. integral solution efficiently!

When is LP=ILP according to the literature?

`ILP` formulation:

$$
\begin{aligned}
f^* &= \min[\mathbf{c}\cdot\mathbf{x}] \\
\text{s.t. } &\mathbf{A}\cdot\mathbf{x} \geq \mathbf{b} \\
&\mathbf{x} \in \mathbb{N}^n/\mathbb{R}^n
\end{aligned}
$$

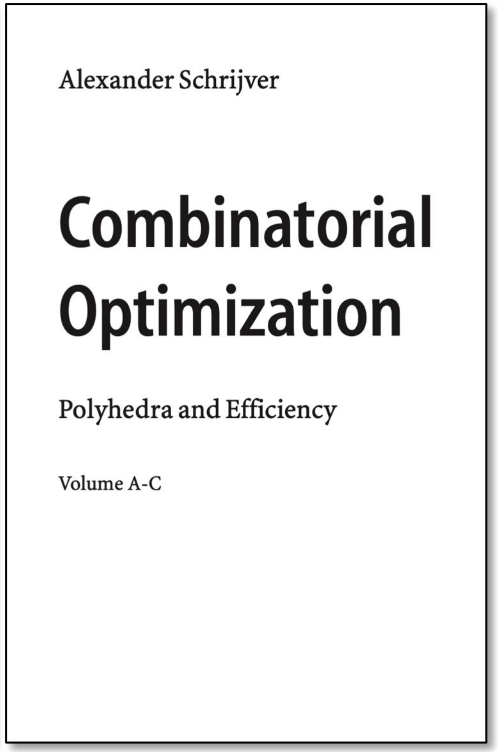




THEORY OF LINEAR AND INTEGER PROGRAMMING

ALEXANDER SCHRIJVER

WILEY–INTERSCIENCE SERIES IN DISCRETE MATHEMATICS AND OPTIMIZATION



Alexander Schrijver

Combinatorial Optimization

Polyhedra and Efficiency

Volume A-C

When is LP=ILP according to the literature: not useful ☹

`ILP` formulation:

objective vector

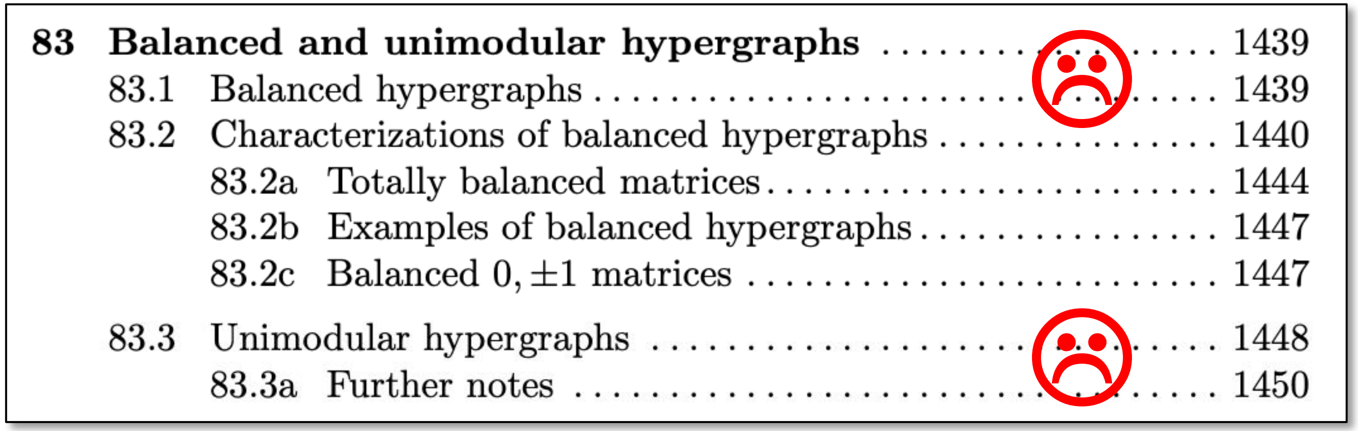
$$f^* = \min[\mathbf{c} \cdot \mathbf{x}]$$

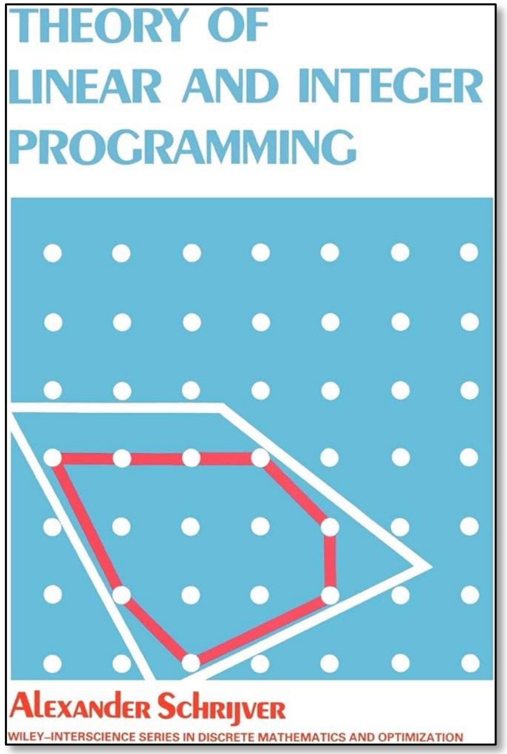
constraint vector

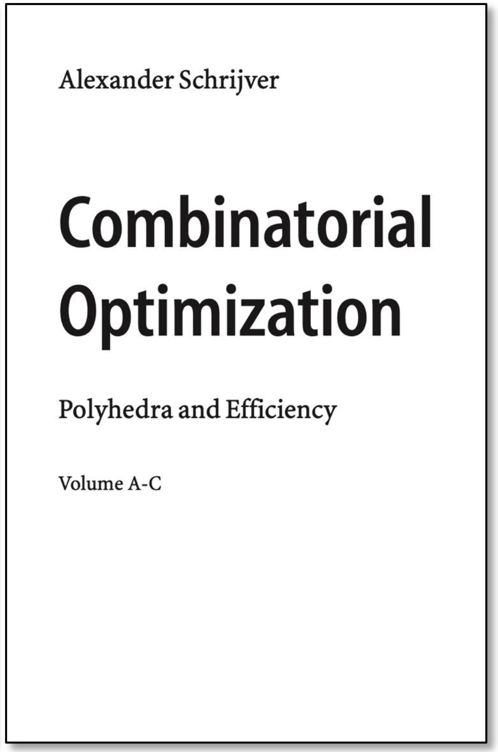
$$\text{s.t. } \mathbf{A} \cdot \mathbf{x} \geq \mathbf{b}$$

constraint matrix

$$\mathbf{x} \in \mathbb{N}^n / \mathbb{R}^n$$







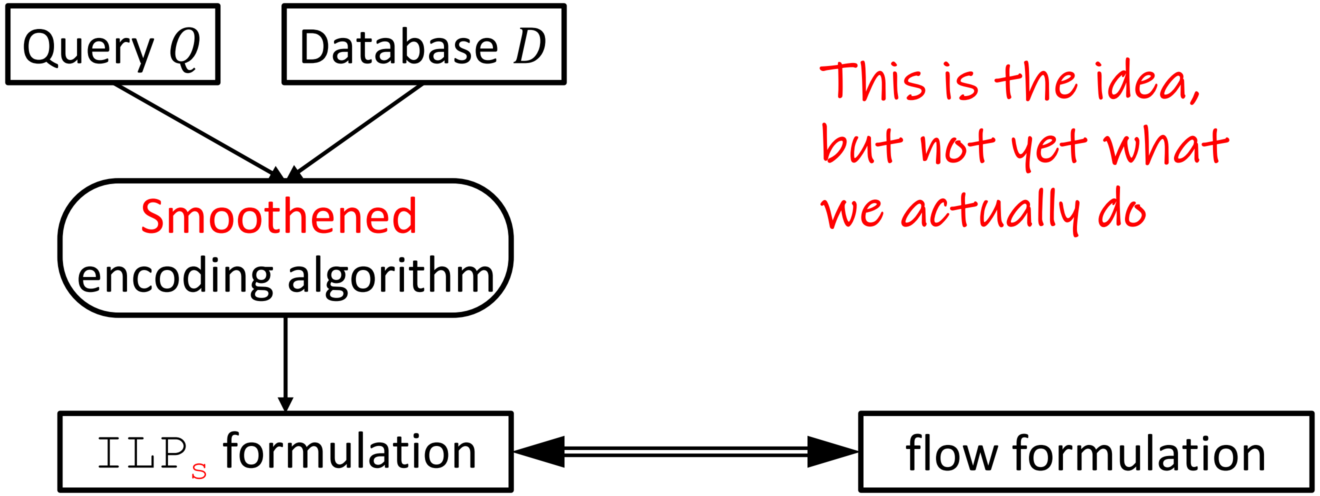
- Focus of polyhedral theory mainly on constraint matrix $\mathbf{A}$. But our PTIME constraint matrixes need not be balanced, nor Totally Unimodular, etc.

1. Our complexity results take into account the objective vector $\mathbf{c}$!
2. This gives us a separation between the problem under set vs. bag semantics!
3. We use an indirect proof via problem-specific MFMC encodings ☺

So what do we do to show ILP=LP for PTIME cases?

`ILP` formulation:

$$\begin{aligned} f^* &= \min[\mathbf{c}\cdot\mathbf{x}] \\ \text{s.t. } \mathbf{A}\cdot\mathbf{x} &\geq \mathbf{b} \\ \mathbf{x} &\in \mathbb{N}^n/\mathbb{R}^n \end{aligned}$$



Query Q

Database D

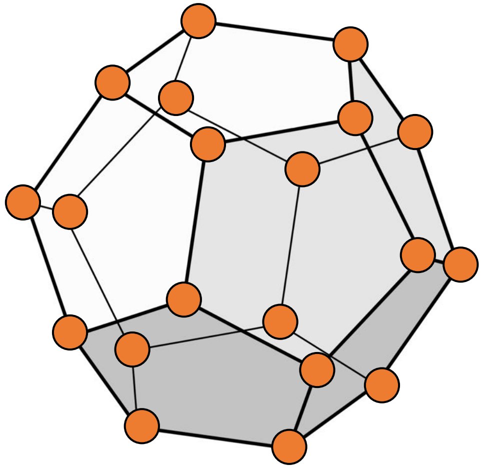
Smoothened encoding algorithm

$\texttt{ILP}_s$ formulation

flow formulation

This is the idea,
but not yet what
we actually do

Polyhedral View



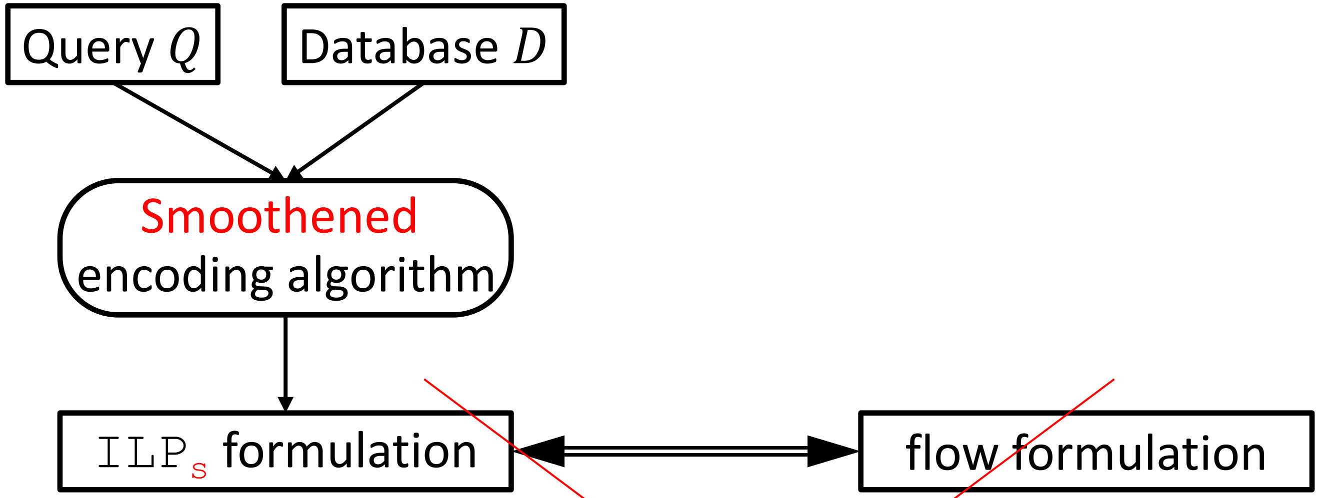
- For an "ideal" constraint matrix, the vertices (extreme points) of its polytope are all integral

1. Show correspondence to a flow formulation
2. Constraint matrix represented by a flow graph is an "ideal matrix"
3. COROLLARY: LP = ILP, PTIME ☺

So what do we do to show ILP=LP for PTIME cases?

`ILP` formulation:

$$f^* = \min[\mathbf{c} \cdot \mathbf{x}]$$
$$\text{s.t. } \mathbf{A} \cdot \mathbf{x} \geq \mathbf{b}$$
$$\mathbf{x} \in \mathbb{N}^n / \mathbb{R}^n$$



Query Q

Database D

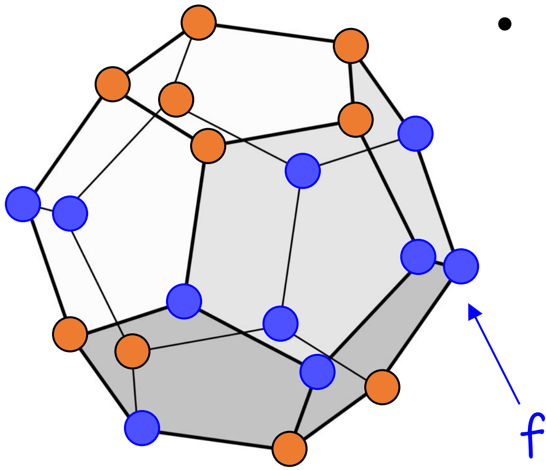
Smoothed encoding algorithm

$\texttt{ILP}_{\texttt{s}}$ formulation

flow formulation

1. ~~Show correspondence to a flow formulation~~
2. ~~Constraint matrix represented by a flow graph is an "ideal matrix"~~
3. ~~COROLLARY: LP = ILP, PTIME ☺~~

Polyhedral View



- For an "ideal" constraint matrix, the vertices (extreme points) of its polytope are all integral

But it is not as easy!

We have fractional extremal points ☹

So what do we do to show ILP=LP for PTIME cases?

`ILP` formulation:

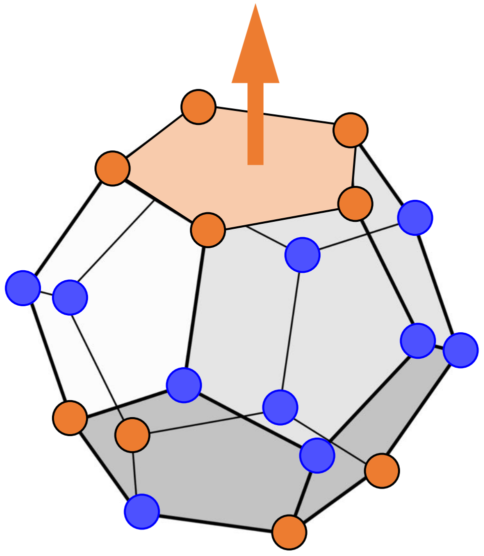
objective vector

$$f^* = \min[\mathbf{c} \cdot \mathbf{x}]$$

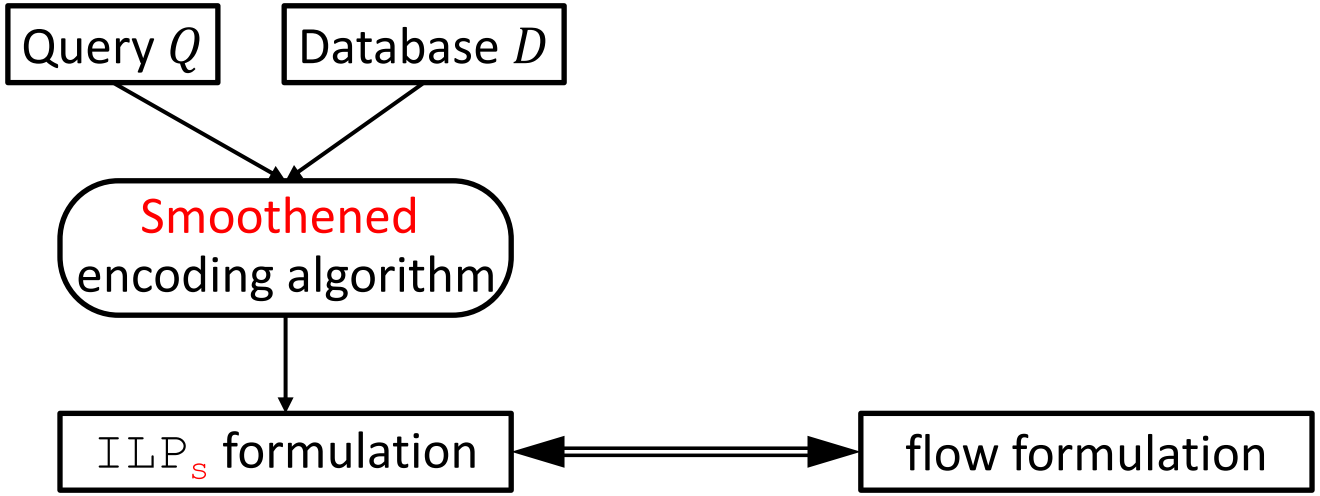
$$\text{s.t. } \mathbf{A} \cdot \mathbf{x} \geq \mathbf{b}$$

$$\mathbf{x} \in \mathbb{N}^n / \mathbb{R}^n$$

Polyhedral View



- The optimal solution face contains only integral vertices
- Polytope may have non-integral vertices



Query Q

Database D

Smoothed encoding algorithm

`ILP`$_{\text{s}}$ formulation

flow formulation

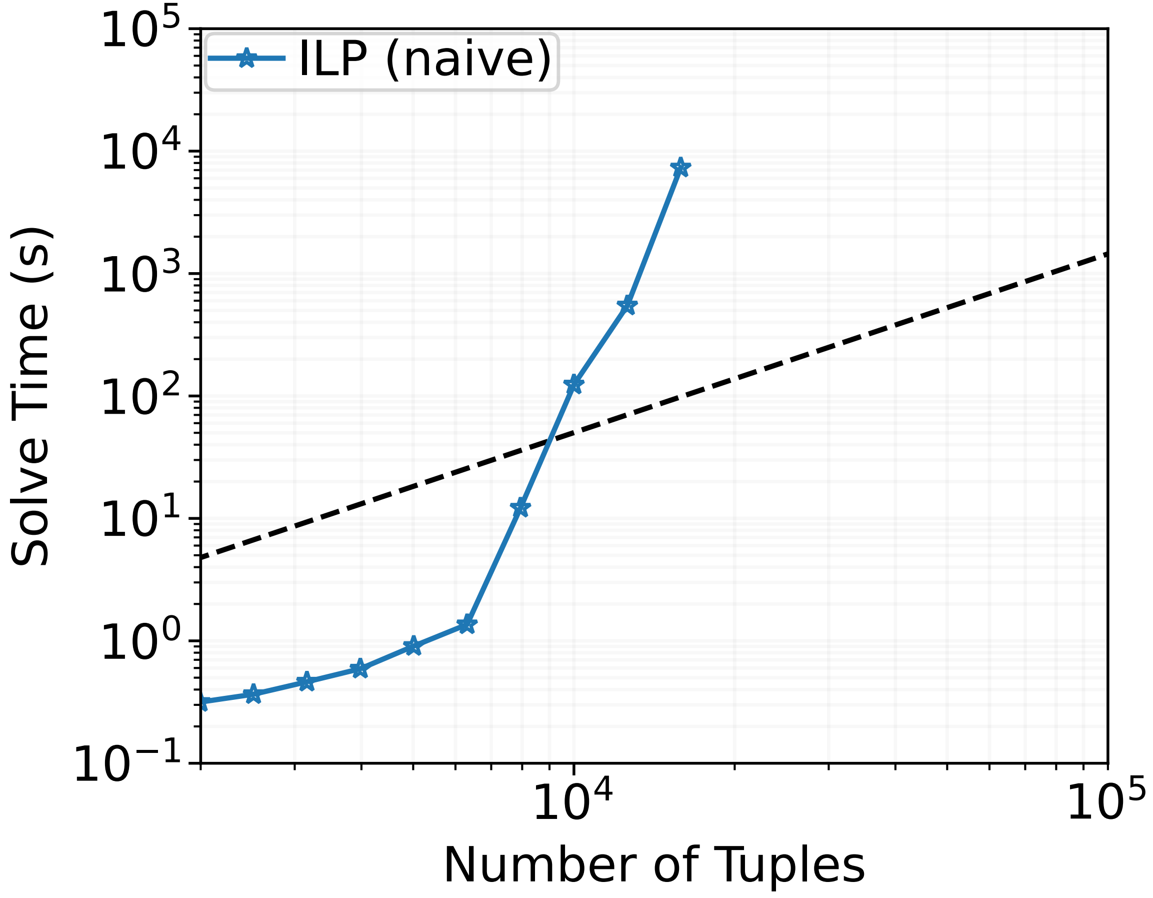
1. Show correspondence *after processing* that takes the objective vector into account
2. *After processing*, constraint matrix is ideal
3. COROLLARY: LP = ILP, PTIME ☺

- Showing such correspondences for all PTIME cases in all scenarios is non-trivial
- This moves the challenge from algorithm development to proofs!

Scalability of Naive vs. Smoothened ILP for PTIME query

Finding an optimal solution for the Smallest Witness Problem ([SIGMOD'19], [ICDT'24])

$Q_{5\text{star}}(x){:}-R(x,a),S(x,b),T(x,c),U(x,d),V(x,e),A(x)$



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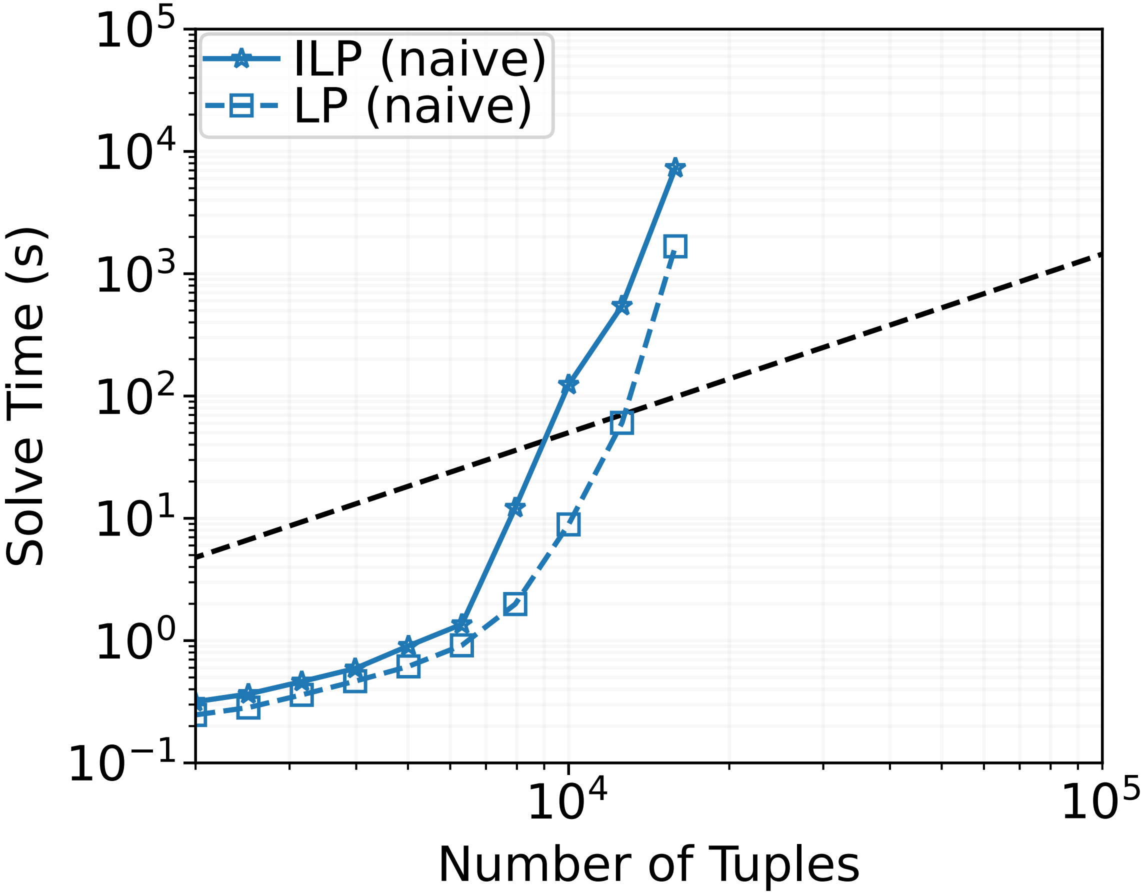
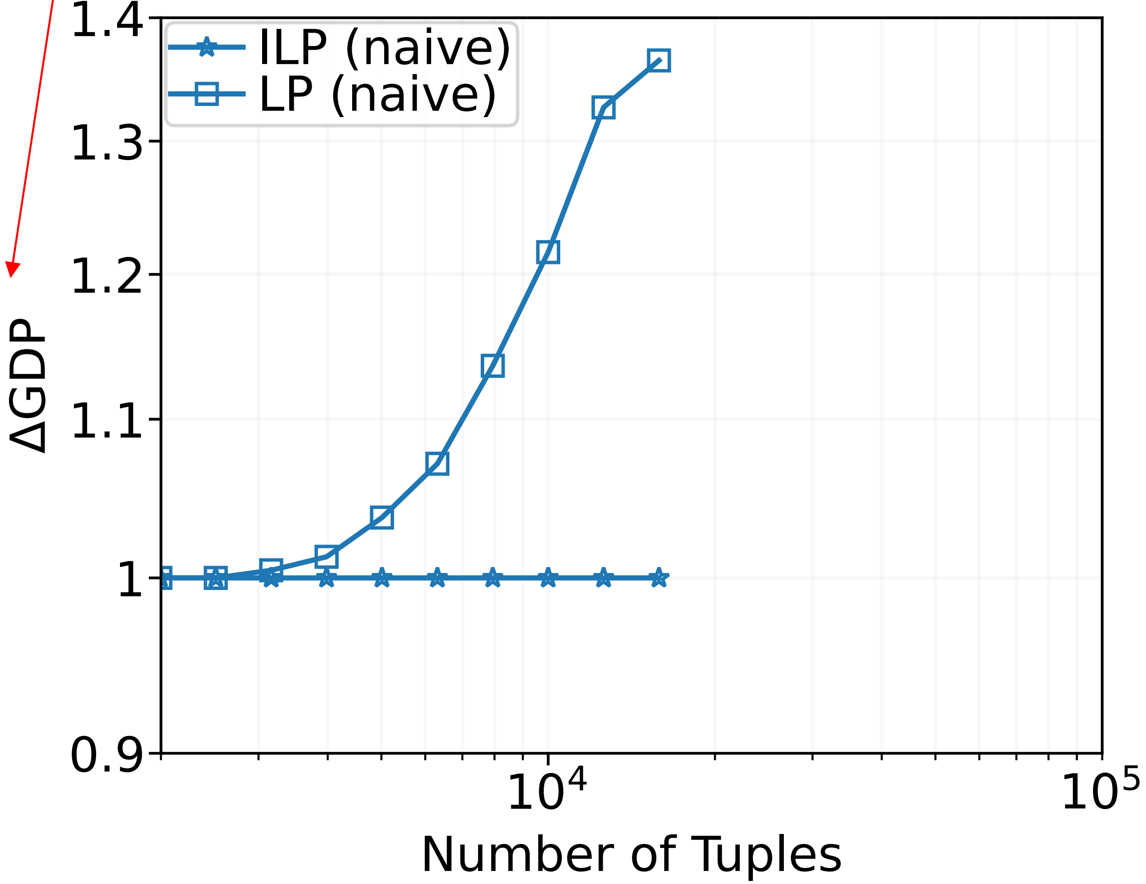
$Q_{5\text{star}}(x){:}-R(x,y),S(x,z),T(x,u),U(x,v),V(x,w),W(x)$



How different are the optimal values for the ILP formulation and its LP relaxation



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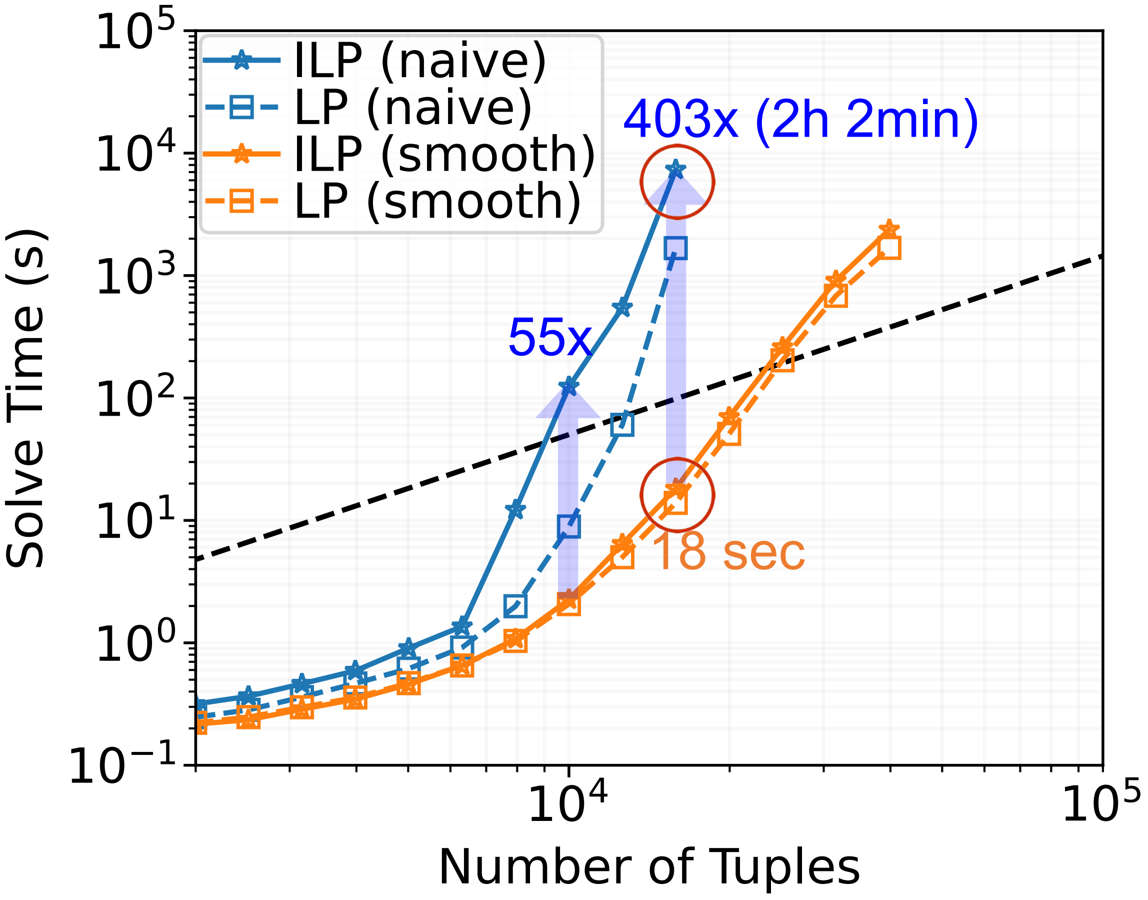

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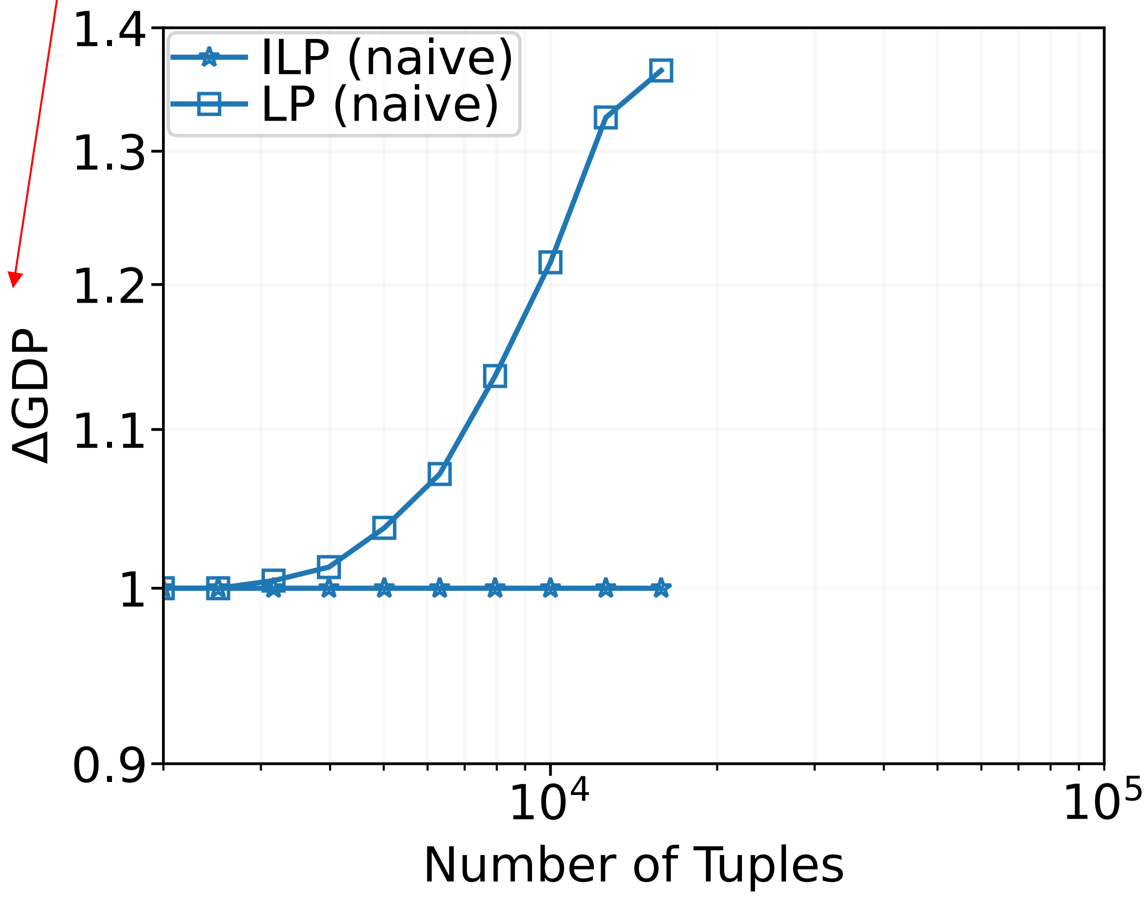

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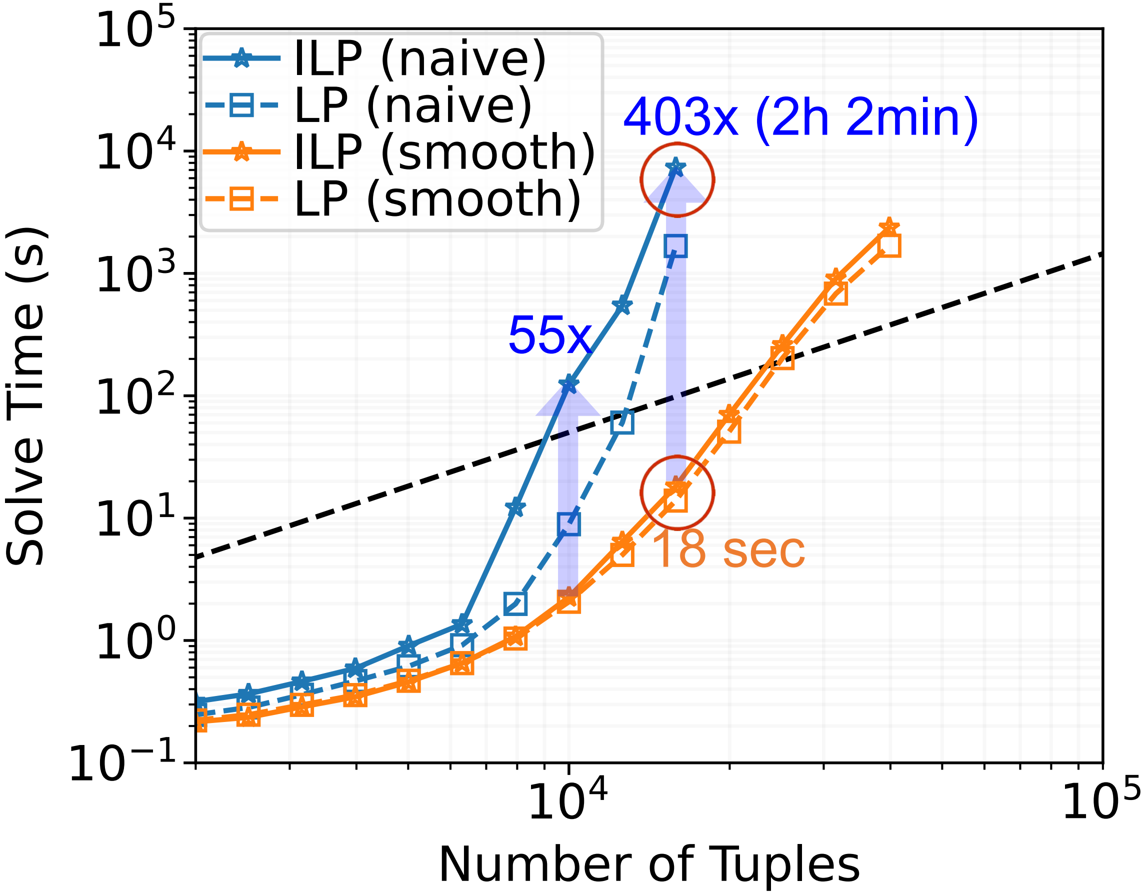

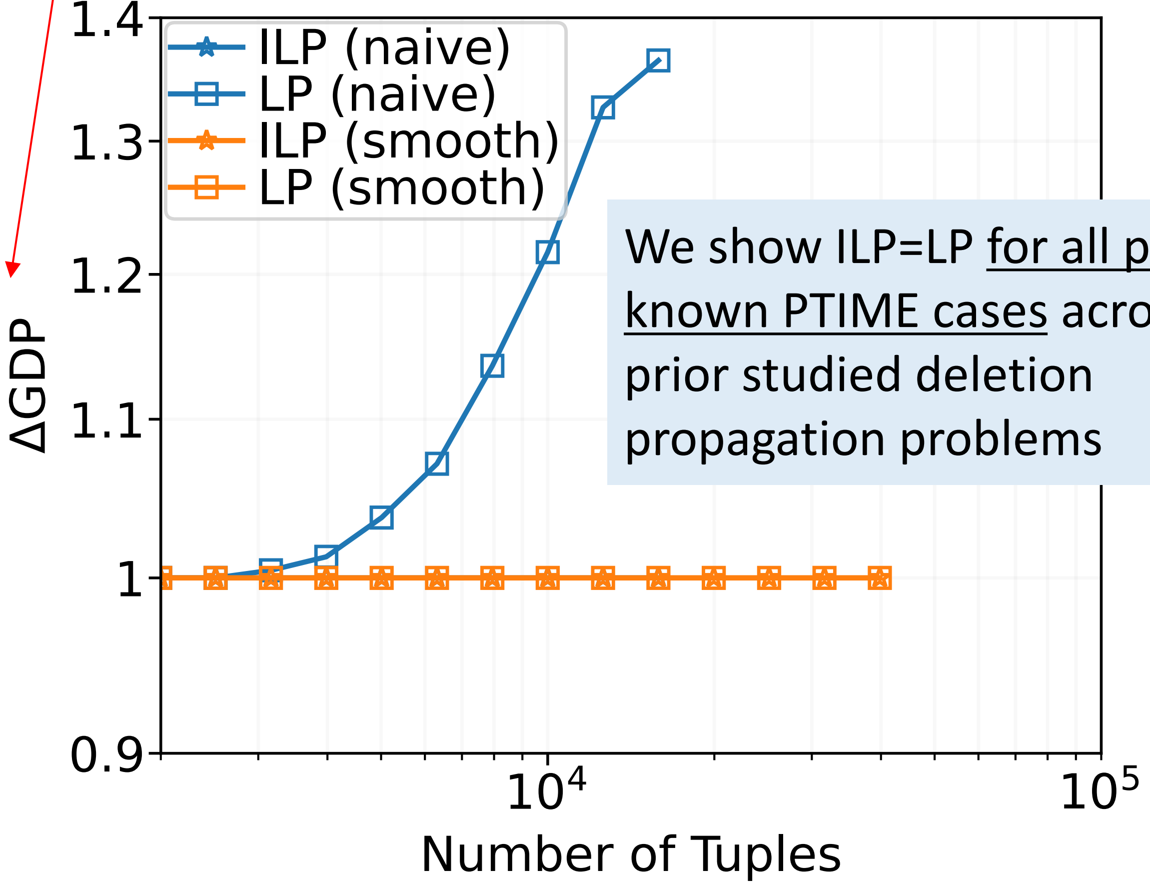

We show ILP=LP for all prior known PTIME cases across prior studied deletion propagation problems

Scalability of Naive vs. Smoothened ILP for PTIME query

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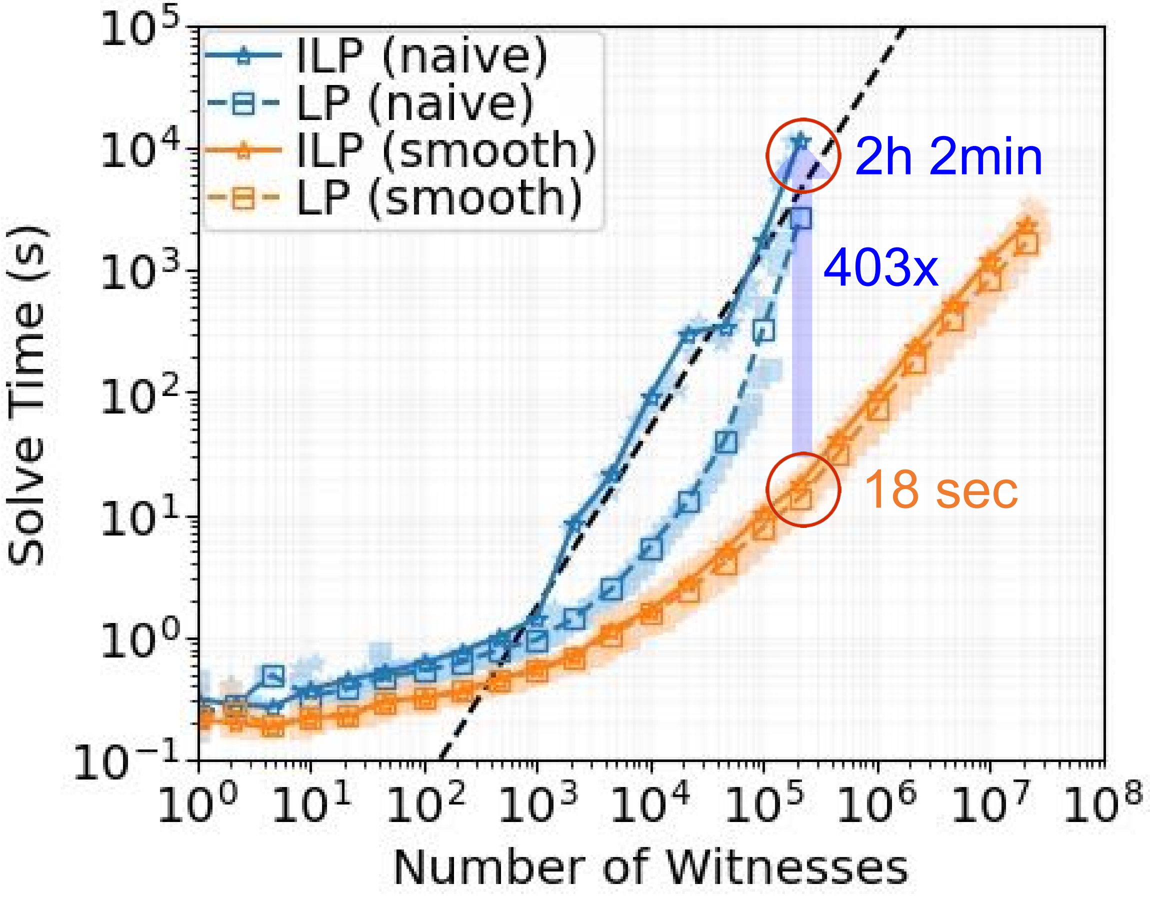

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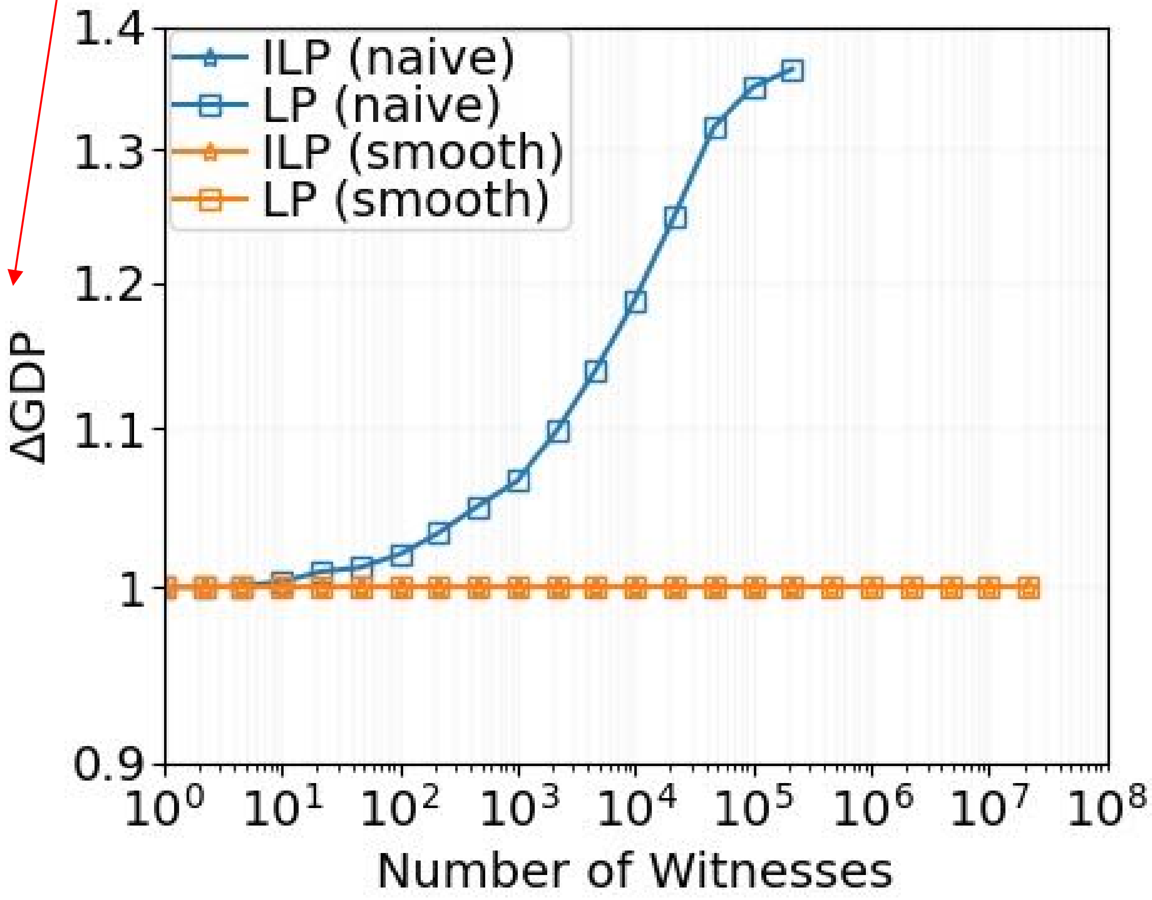
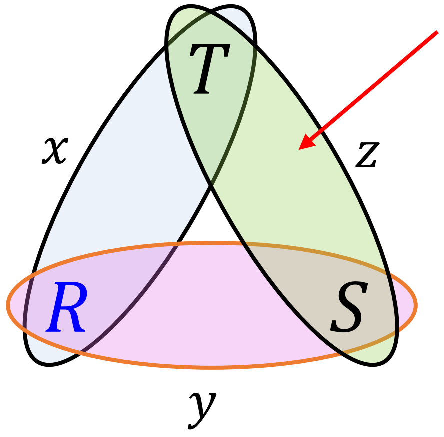


Example complicated landscape for resilience

Triangle query

$Q^{\Delta}:-R(x,y),S(y,z),T(x,z)$



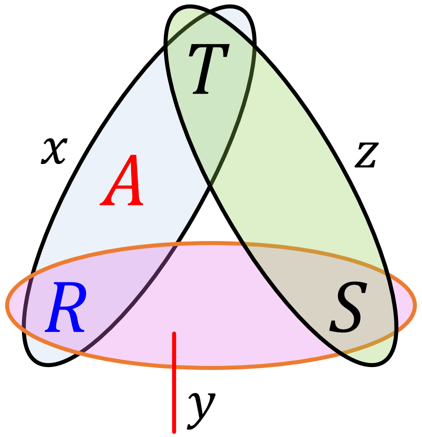
Dual Hypergraph

```
select exists(
   select 1
   from R, S, T
   where R.y=S.y
   and S.z=T.z
   and T.x=R.x)
```

NPC

Triangle unary

$Q_{A}^{\Delta}:-R(x,y),S(y,z),T(x,z),A(x)$



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PTIME

Freire, Gatterbauer, Immerman, Meliou. The complexity of resilience and responsibility for self-join-free conjunctive queries, VLDB 2015. https://doi.org/10.14778/2850583.2850592

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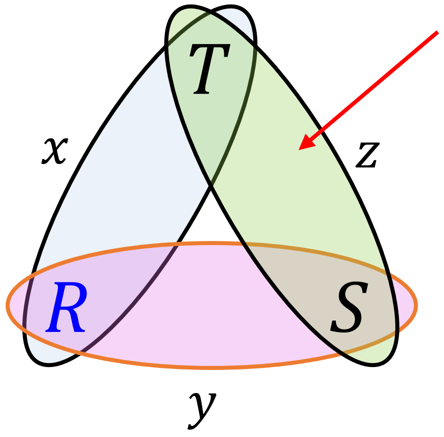

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NPC

PTIME for FD $x \rightarrow y$

Triangle unary

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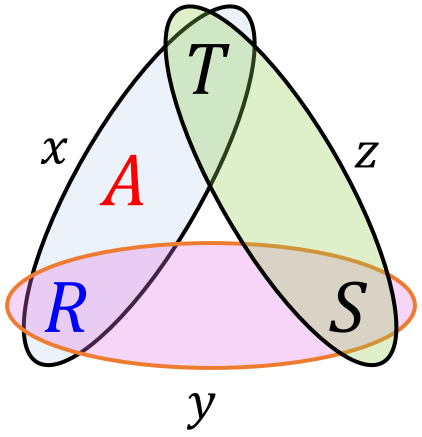

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PTIME

NPC under bag semantics

PTIME if provenance happens to be read-once

Freire, Gatterbauer, Immerman, Meliou. The complexity of resilience and responsibility for self-join-free conjunctive queries, VLDB 2015. https://doi.org/10.14778/2850583.2850592
Makhija, Gatterbauer. A Unified Approach for Resilience and Causal Responsibility with Integer Linear Programming (ILP) and LP Relaxations, SIGMOD 2024. https://doi.org/10.1145/3626715

Example complicated landscape for resilience

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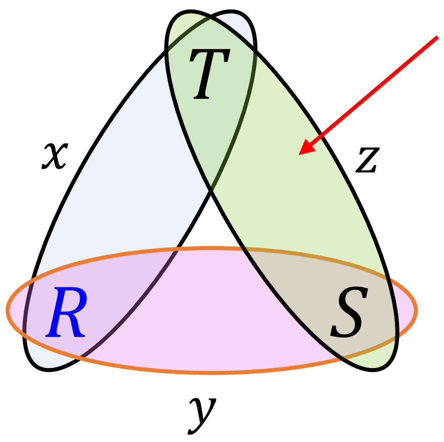

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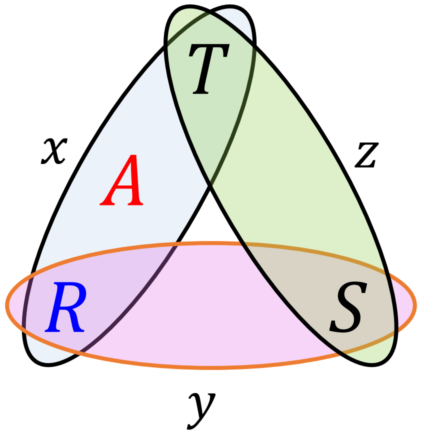

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PTIME

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"Coarse-grained instance-optimal" algorithm

Freire, Gatterbauer, Immerman, Meliou. The complexity of resilience and responsibility for self-join-free conjunctive queries, VLDB 2015. https://doi.org/10.14778/2850583.2850592
Makhija, Gatterbauer. A Unified Approach for Resilience and Causal Responsibility with Integer Linear Programming (ILP) and LP Relaxations, SIGMOD 2024. https://doi.org/10.1145/3626715
Makhija, Gatterbauer: Is ILP all you need for Deletion Propagation? PVLDB 2025.

Outline

1. Reverse Data Management (RDM)
2. A magical ILP formulation
3. Take-aways

(An illustrated example: only if time remains)

A possible shift of focus of algorithm design in database theory?

Current focus: discrete algorithms

Identify tractable cases for a class of problems that can be solved with some dedicated discrete algorithm (like dynamic programming or greedy) or a reduction to flow

For the hard cases:

- prove hardness via some dedicated reduction from some NPC problem.
- optionally design a separate dedicated approximation algorithm

Partial solutions: Often, the algorithm (the dichotomy) does not extend to all types of queries like CQs with self-joins, or problems under bag semantics

Future: polyhedral algorithms

Design one "appropriate" ILP program to solve all problems

- "appropriate" here means that their natural LP relaxation has the same optimal objective for all PTIME cases ("LP=ILP"), which proves the ILP can be solved in PTIME.

All cases are covered (including the hard ones) ✓

- also approximation algorithms, just stop evaluation early, anytime algorithm comes for free ✓

Complete solutions: All problem types are covered ✓ (including self-joins or bag semantics)

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An anonymous concern: "...the ILP constructed is not a simple mathematical object... Since the construction given is .... not a simple mathematical object, it is not clear to me how deep one can push this further by analyzing it."

A possible shift of focus of algorithm design in database theory?

	Current focus: discrete algorithms	**Future: polyhedral algorithms**
Theory	Identify tractable cases for a class of problems that can be solved with some dedicated discrete algorithm (like dynamic programming or greedy) or a reduction to flow	Design one "appropriate" ILP program to solve all problems • "appropriate" here means that their natural LP relaxation has the same optimal objective for all PTIME cases ("LP=ILP"), which proves the ILP can be solved in PTIME.
	For the hard cases: • prove hardness via some dedicated reduction from some NPC problem. • optionally design a separate dedicated approximation algorithm	All cases are covered (including the hard ones) ✓ • also approximation algorithms, just stop evaluation early, anytime algorithm comes for free ✓
	Partial solutions: Often, the algorithm (the dichotomy) does not extend to all types of queries like CQs with self-joins, or problems under bag semantics	Complete solutions: All problem types are covered ✓ (including self-joins or bag semantics)
	Practical aspects: usually only some problem cases are solved, hard cases often not treated, the practical nature of the algorithms is not always clear	**Practical aspects**: it works from day one ✓

Take-aways: "Is ILP all you need? ..."

- Polyhedral theory solves many database theory problems "out of the box".
  - Shift in focus: instead of trying to find a dedicated PTIME algorithm for PTIME cases, start with a general formulation and prove it finishes in PTIME for PTIME cases.
  - There is some magic in getting the "right" formulation (ILP = LP for PTIME), we don't yet have "the" recipe
  - The proofs for LP=ILP go beyond standard optimization literature. Polyhedral theory alone does not help.
- The overall philosophy is way more general than reverse data management.
  - Makhija, Gatterbauer. *Minimally Factorizing the Provenance of Self-Join Free Conjunctive Queries*, PODS 2024. https://doi.org/10.1145/3651605
  - What about consistent query answering? And even more general logic optimization problems?
- More concretely open: Unifying deletion and insertion propagation ("change propagation"), basically positive and negative provenance / Why or Why not?
  - Meliou, Gatterbauer, Moore, Suciu. *Why so? or Why no? Functional causality for explaining query answers*. MUD 2010. https://arxiv.org/pdf/0912.5340
- Please also talk to Neha ☺
  Faculty at UMass Amherst from Fall'25

Thank you ☺