

Counterexamples to commonly held Assumptions on Unit Commitment and Market Power Assessment

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- Comparison of Theoretical Efficiency of Centralized and Decentralized Unit Commitment (PoolCo vs. PX)
- Determination of Market Power revisiting the fundamental Economic Assumption of Marginal Costs being the baseline of competitive prices

Agenda 1: PoolCo vs PX



- Background Information
- The commonly used Argument
- Counterexample
- Conclusions

Background Information 1: PoolCo vs PX



- Unit Commitment: Technological constraints (minimum up-time, starting costs)
- ISO: Independent System Operator
- PoolCo vs. PX (Power Exchange)

Agenda 1: PoolCo vs PX



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Conventional Centralized Unit Commitment



- Minimize the total generation cost

$$\min_{u_i, Q_i} \sum_{i=1}^n u_i C_i(Q_i)$$

- So that total generation equals total load

$$\sum_{i=1}^n Q_i = Q_D$$

- Lagrangian relaxation method

$$L(u, Q, I) = \sum_{i=1}^n u_i (C_i(Q_i) - I Q_i) + I Q_D$$

Conventional Centralized Unit Commitment



- Minimized over Q

$$\frac{dC_1}{dQ_1} = \dots = \frac{dC_n}{dQ_n} = \lambda$$

- Plug back

$$L(u, \lambda) = \sum_{i=1}^n u_i (C_i(Q_i(\lambda)) - \lambda Q_i(\lambda)) + \lambda Q_D$$

- Minimized with respect to u_i -> Switching Law

$$u_i = \begin{cases} 0 & \text{if } C_i - \lambda Q_i > 0 \\ 1 & \text{if } C_i - \lambda Q_i < 0 \end{cases}$$

$$\boxed{\frac{C_i}{Q_i} < \lambda}$$

Decentralized Unit Commitment



- Maximize the individual profit

$$\max_{Q_i} \mathbf{p}_i(Q_i) \quad \mathbf{p}_i = P Q_i - C_i(Q_i)$$

- Decide in advance whether to turn on the unit

$$u_k \quad \hat{Q}_k \quad \hat{p}_k$$

- Expected Profit

$$\hat{\mathbf{p}}_{on} = \hat{p} \cdot \hat{Q}_i - C_i(\hat{Q}_i)$$



- Decision

$$\hat{p}_{on} > 0$$

- Switching Law

$$\frac{C_i(\hat{Q}_i)}{\hat{Q}_i} < \hat{p}$$

- Conclusion: a centralized system operator would schedule the same units as the individual power producers would in a decentralized way

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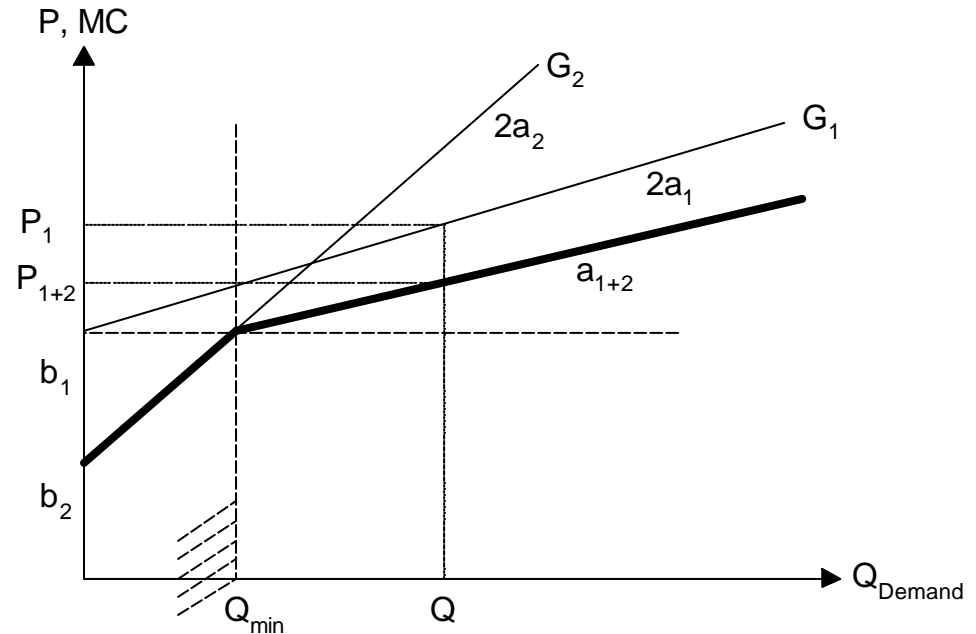
Counterexample: 2 Generators G_1, G_2



- Quadratic Cost Function: $C_i(Q_i) = a_i Q_i^2 + b_i Q + c_i$
- Linear increasing MC: $MC_i(Q_i) = 2a_i Q_i + b_i$

- Supply Functions:

- $Q_i = f(a, b, c, Q)$



Counterexample: Conditions



We search Parameters a, b, c, Q so as to:

- Generator 1 makes profits:

$$a_1 Q_1^2 + b_1 Q_1 + c_1 < P_{1+2} Q_1$$

- Generator 2 loses money if switched on:

$$a_2 Q_2^2 + b_2 Q_2 + c_2 > P_{1+2} Q_2$$

- Total costs are lower with both generators on:

$$a_1 Q^2 + b_1 Q + c_1 > a_1 Q_1^2 + b_1 Q_1 + c_1 + a_2 Q_2^2 + b_2 Q_2 + c_2$$

Counterexample: Numerical Values



- Typical Parameters:

	G_1	G_2
a	1	2
b	1	1.6
c	1.1	0.7
Q	2	

- Differences:

	G_1	G_1 and G_2		
P	5	3.87		
	G_1	$G_1 + G_2$	G_1	G_2
$\% Q$	100%	100%	72%	28%
C	7.1	6.84	4.59	2.25
Rev	10	7.73	5.54	2.19
\mathcal{R}	2.9	0.9	0.95	-0.06

Agenda 1: PoolCo vs PX



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Conclusion 1 (PoolCo vs PX)



- A centralized Unit Commitment can lead to higher efficiency
- Explanation: It is possible that several generators can supply the demand with lower costs than the subgroup of generators that would obtain a profit in a free competitive market
 - assuming bidding marginal costs (!)

Agenda 2: Market Power



- Background Information
- Illustrative Example
- Numerical Values
- Conclusions



- „Offering power at a price significantly above marginal production (or opportunity) cost, or failing to generate power that has a production cost below the market price, is an indication of the exercise of market power...“ [Borenstein00]
- “Market power exists when a supplier or consumer influences prices ... If suppliers exercise market power, prices could be higher than marginal costs.” [DOE97]

Background Information 2 (Market Power)



- „Economic withholding occurs when a supplier offers output to the market at a price that is above both its full incremental costs and the market price (and thus, the output is not sold)” [FERC01]

[Borenstein00]: Borenstein S., Bushnell J., Wolak F.; Diagnosing Market Power in California's Restructured Wholesale Electricity Market; NBER Working Paper 7868

[DOE97]: Department of Energy; Electricity Prices in a Competitive Environment: Marginal Cost Pricing of Generation Services and Financial Status of Electric Utilities.

[FERC01]: Federal Energy Regulatory Commission; Investigation of Terms and Conditions of Public Utility Market-Based Rate Authorizations; Order E-47,

Agenda 3: Market power

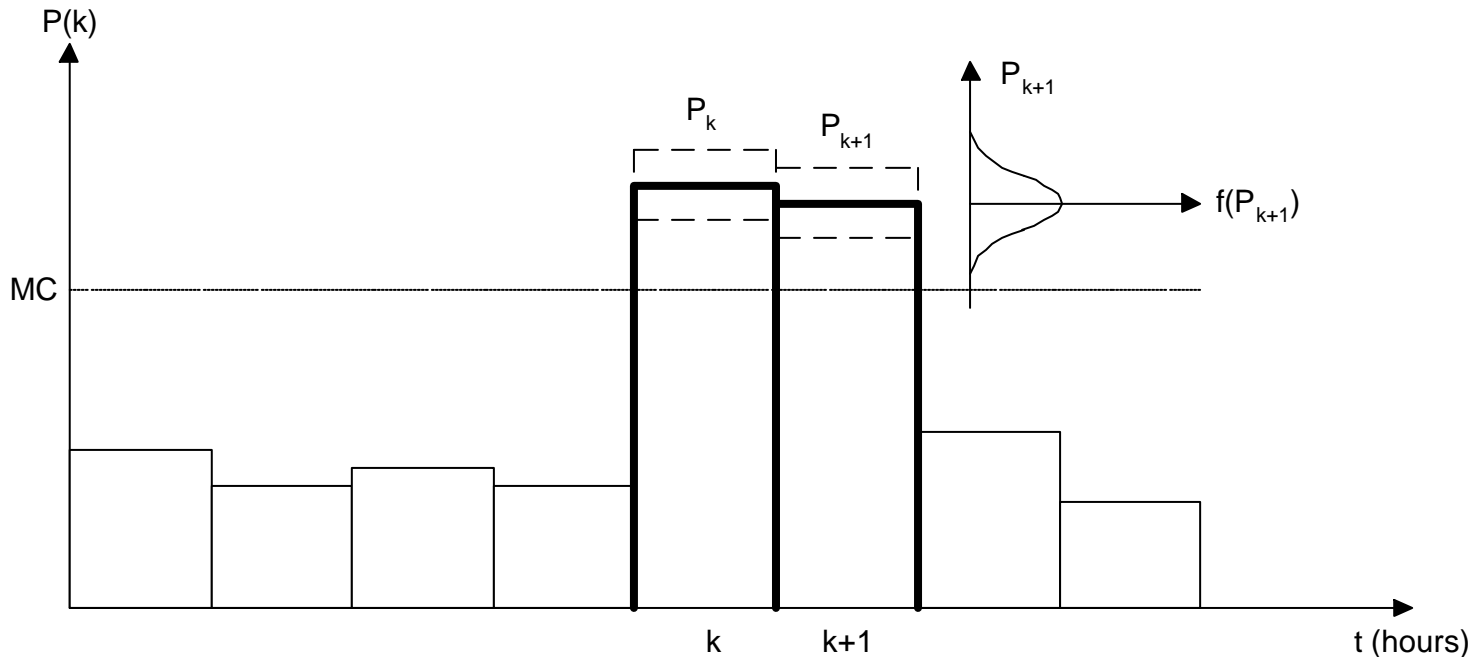


- Background Information
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- Conclusions

Illustrative Example



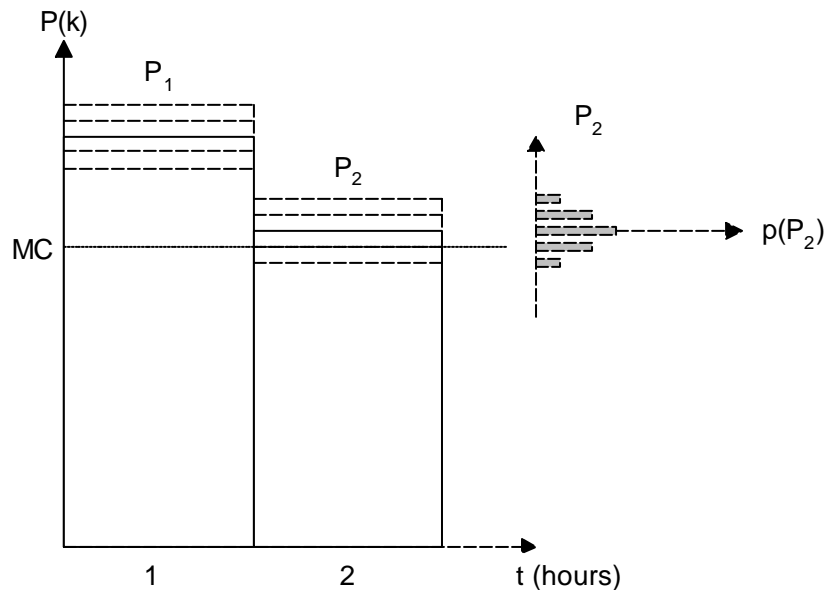
- $MC = \text{const}$, $t_{\text{up},\text{min}} = 2$, $SU + SD = \text{FOC}$
- Price taker



Illustrative Example



- Discrete Prices: $P_1 \in \{P_{11}, \dots, P_{1i}, \dots, P_{15}\}$
 $P_2 \in \{P_{21}, \dots, P_{2j}, \dots, P_{25}\}$

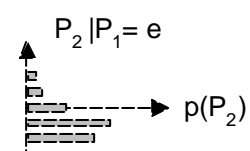
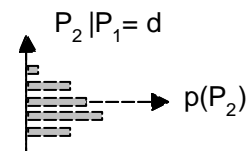
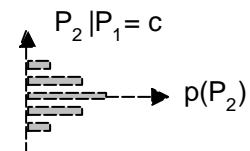
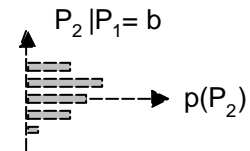
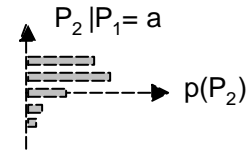
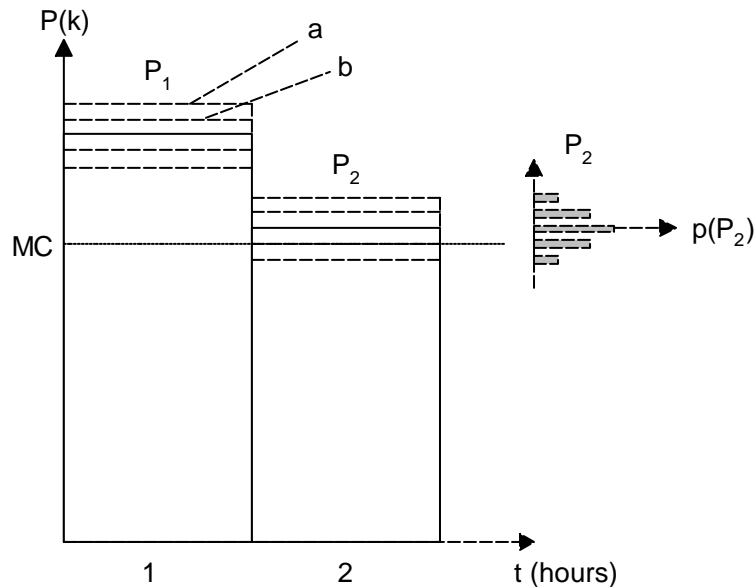


Illustrative Example



- Correlation between Hours possible

$$p(P_2 = P_{2j}) = p(P_2 = P_{2j} | R_1 = R_{1i})$$



Agenda 3: Market Power



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Numerical Values – Example



- $MC=50, Q=1, FOC=10$

	Bid Sequence	Exp.Profit
Prices independent	(58,52), (60,54)	1.1720
	(58,54)	1.1538
	(50,50)	1.0798
	(56,50),(56,52), (60,56),(62,56)	
	(60,52)	0.9266
Prices correlated	(60,52)	1.7650
	(58,52)	1.6838
	(60,54)	1.6834
	(50,50)	1.0798

Agenda 3: Market Power



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Conclusion 2 (Market Power)



- Market Prices above MC of the last unit do not prove the exercise of Market Power (!)
- In order to determine the optimal bidding sequence, the price correlations between hours have to be included in the algorithms



- A decentralized Unit Commitment is not always as efficient as the centralized one – even in the theoretical case.
- Marginal Costs cannot be used as the baseline from which Market Power is measured.

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Backup – 3 – Formula

- Expected Profit of Bidding (b_1, b_2):

$$\begin{aligned} J(b_1, b_2) = & \sum_{P_i | P_i \geq b_1} \sum_{P_j | P_j \geq b_2} p(P_1 = P_i) \cdot p(P_2 = P_j) \cdot \left(\begin{array}{l} (P_i + P_j - 2MC)Q \\ -FOC \end{array} \right) \\ & + \sum_{P_i | P_i \geq b_1} \sum_{P_j | P_j < b_2} p(P_1 = P_i) \cdot p(P_2 = P_j) \cdot ((P_i - MC)Q - FOC) \\ & + \sum_{P_i | P_i < b_1} \sum_{P_j | P_j \geq b_2} p(P_1 = P_i) \cdot p(P_2 = P_j) \cdot ((P_j - MC)Q - FOC) \end{aligned}$$

Backup – 3 – Numerical Values

$$MC=50; \quad P_1 \in \{56, 58, 60, 62, 64, 66\}$$

$$Q=1; \quad P_2 \in \{46, 48, 50, 52, 54, 56\}$$

$$FC=10;$$

With $p_i = p(P_1 = P_{1i}) = p(P_2 = P_{2i})$ and $p_{i|j} = p(P_2 = P_{2j} | P_1 = P_{1i})$:

$p_1=0.1888$	$p_{1 1}=0.45$	$p_{1 2}=0.20$	$p_{j 3} = p_j$
$p_2=0.1624$	$p_{2 1}=0.20$	$p_{2 2}=0.32$	
$p_3=0.2978$	$p_{3 1}=0.27$	$p_{3 2}=0.33$	$p_{j 4} = p_{5-j 2}$
$p_4=0.1624$	$p_{4 1}=0.06$	$p_{4 2}=0.08$	
$p_5=0.1888$	$p_{5 1}=0.02$	$p_{5 2}=0.08$	$p_{j 3} = p_{5-j 1}$