Optimal Algorithms for Ranked Enumeration of Answers to Full Conjunctive Queries

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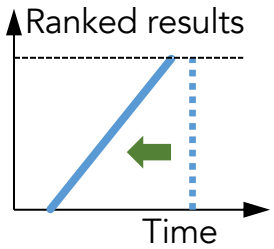
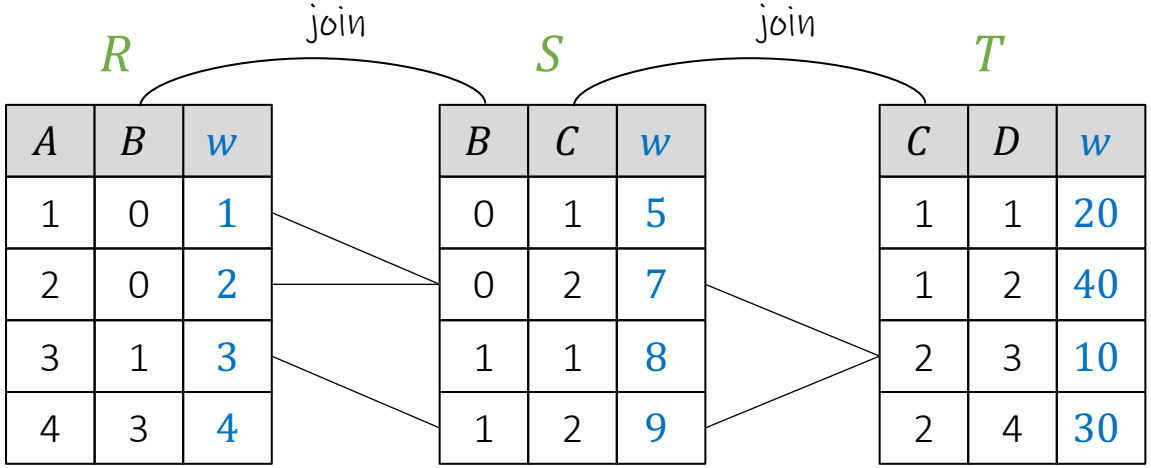


Project Page: https://northeastern-datalab.github.io/anyk/
Data Lab: https://db.khoury.northeastern.edu

Ranked Enumeration Example




```
select   A, B, C, D,
         R.w + S.w + T.w as weight
from     R, S, T
where    R.B=S.B and S.C=T.C
order by weight ASC
limit k  any-k
```

Enumerate results in order

Rank-1 (1, 0, 2, 3, 18) ⇨ Rank-2 (2, 0, 2, 3, 19) ⇨ Rank-3 (3, 1, 2, 3, 22) ⇨ ...

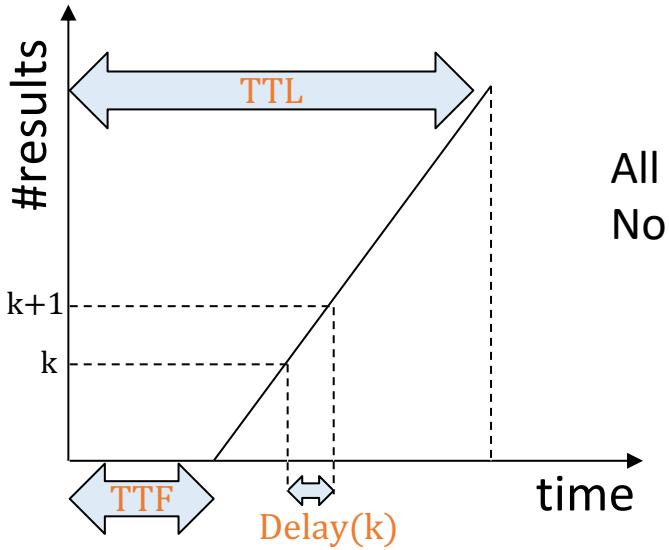
SUM of weights

Ranked Enumeration: Problem Definition

"Any-k"

Anytime algorithms + Top-k for Conjunctive Queries

Most important results first (ranking function on output tuples, e.g. sum of weights)

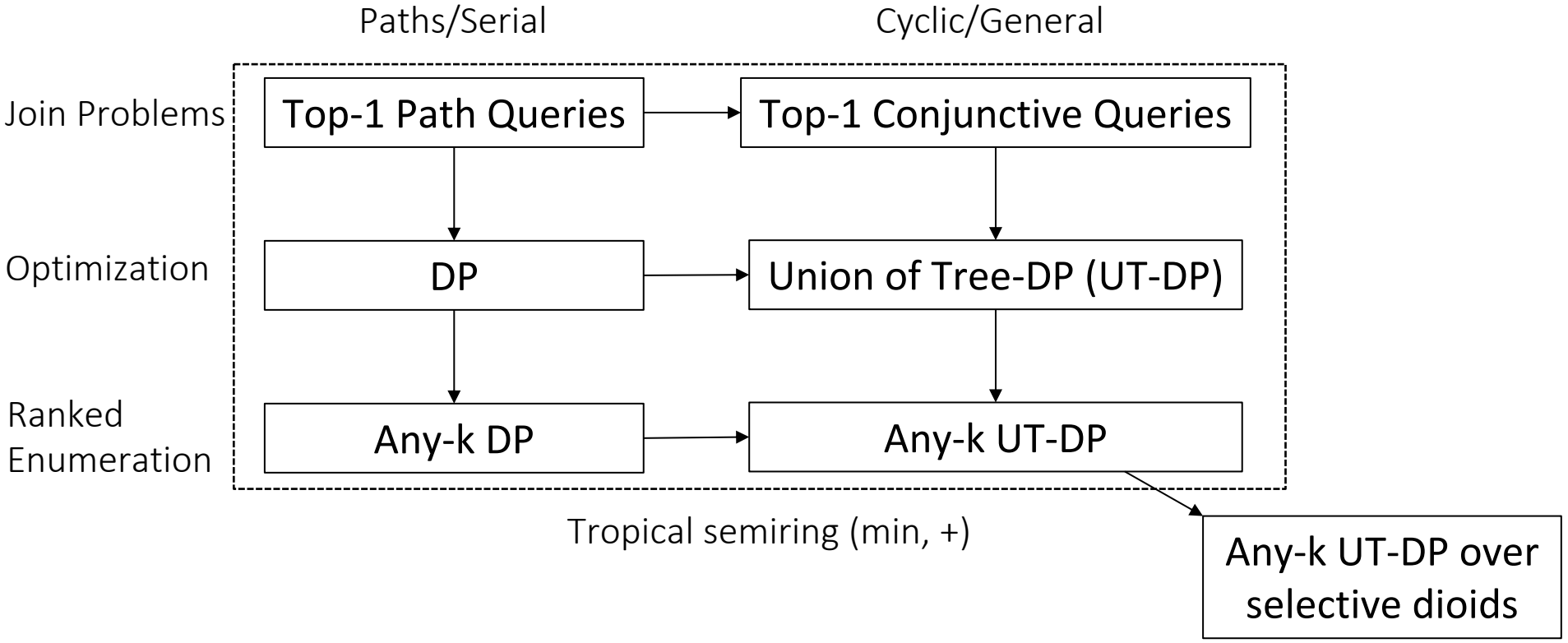


All results eventually returned
No need to set *k* in advance

RAM Cost Model:

- TTF = Time-to-First = TT(1)
- Delay(k) = Time between Rank-k and Rank-(k+1)
- TTL = Time-to-Last = TT(|out|)

Conceptual Roadmap



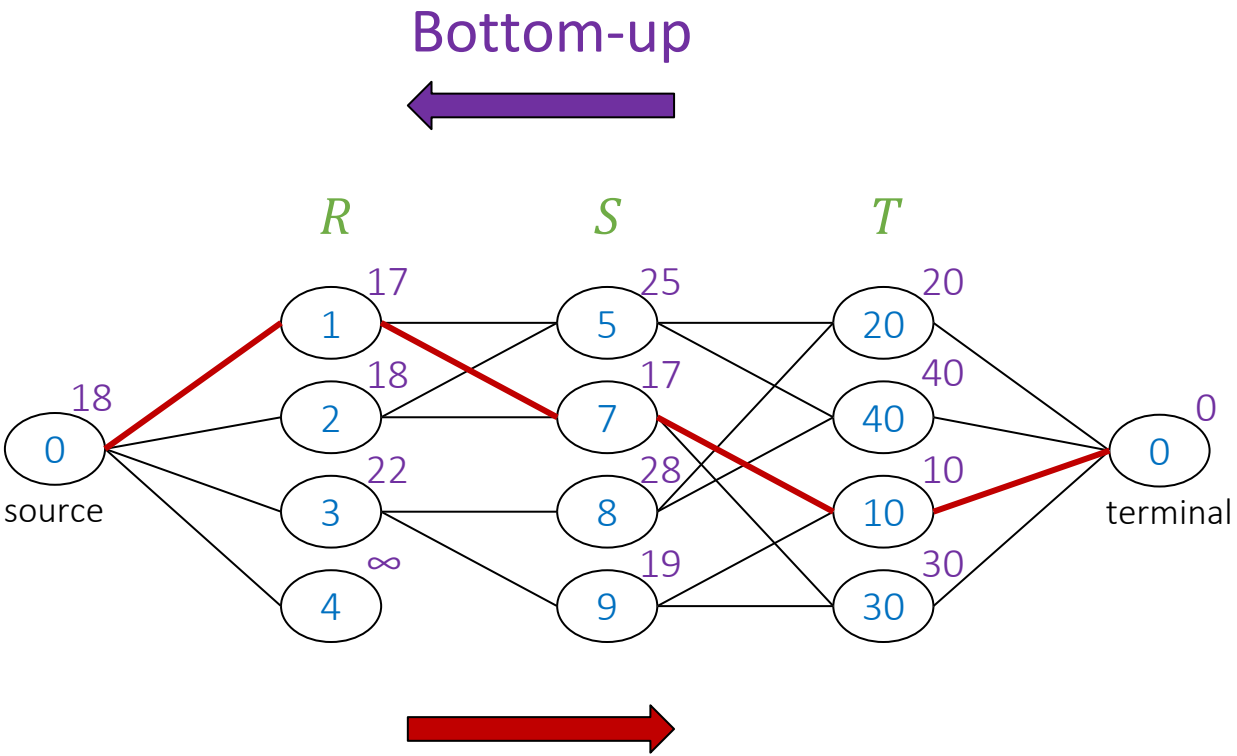
Main Result

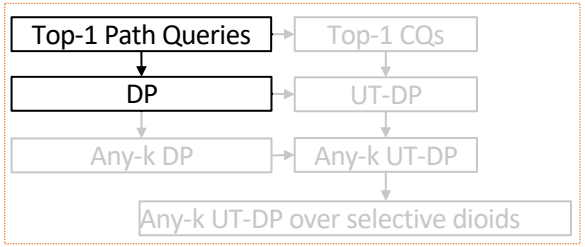
n: database size
Query = fixed size

- For Acyclic Queries:
  - $\mathrm{TTF} = O(n)$
  - Delay(k) = $O(logk)$
  - We get k results (sorted) in just $O(n + k\ \log k)$ for any k!

- For Cyclic Queries:
  - Higher TTF, according to best tree decomposition(s) available
  - Inherent cost of cyclicity

Top-1: Dynamic Programming





Nodes: tuples

Edges: joining pairs

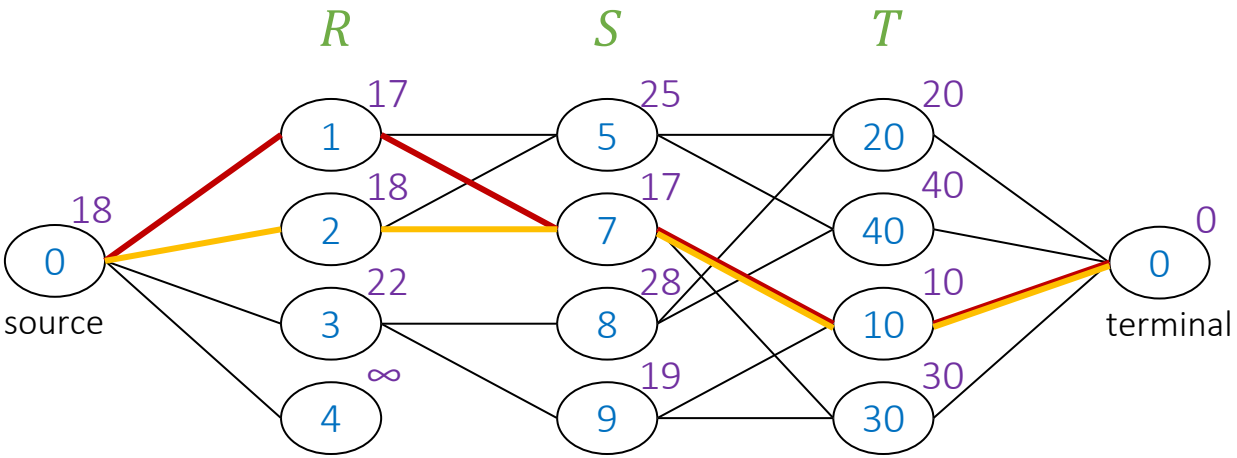
Labels: tuple weights

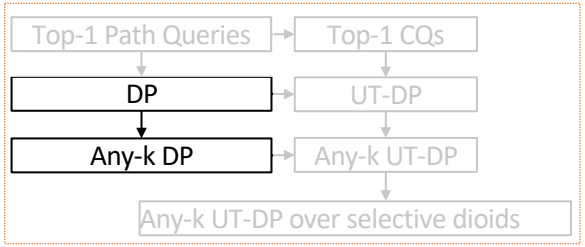
Bottom-up values: min total weight

Paths: join results

Any-k DP: k-shortest paths

2nd Best Result = 2nd Shortest Path (19)





Best result = Shortest Path (18)

Any-k DP Algorithms: 2 non-dominated families

Anyk-Part

Repeatedly partitions the solution space.
Relies on [Lawler MS'76]

Wins when k is small.

Variants
- Eager
- All [Yang+ WWW'18]
- Lazy [Chang+ VLDB'15]
- **Take2** →

n: database size
l: query size

$$\text{TTF} = O(ln)$$
$$\text{Delay(k)} = O(\log k + l)$$

Lowest delay given linear-time pre-processing!

Anyk-Rec

Recursively computes lower-rank paths (suffixes) and reuses them.
Inspired by[Jiménez+ WEA'03]

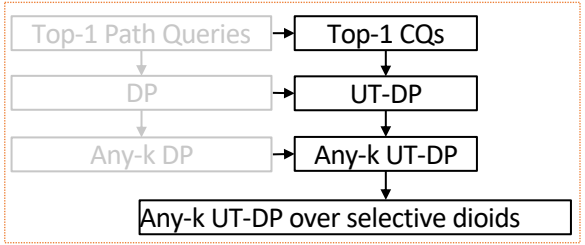
Wins when k is large.

$$\text{TTF} = O(ln)$$
$$\text{Delay(k)} = O(\log n \cdot l)$$

Reusing computation may pay off – can be even faster than sorting!
For Cartesian product with n^ℓ results:
Anyk-Rec TTL: $O\left(n^\ell(\log n + \ell)\right)$
Batch-Sorting/Anyk-Part: $O\left(n^\ell \log n \cdot \ell\right)$

Generalizations

- Paths → Trees (Acyclic)

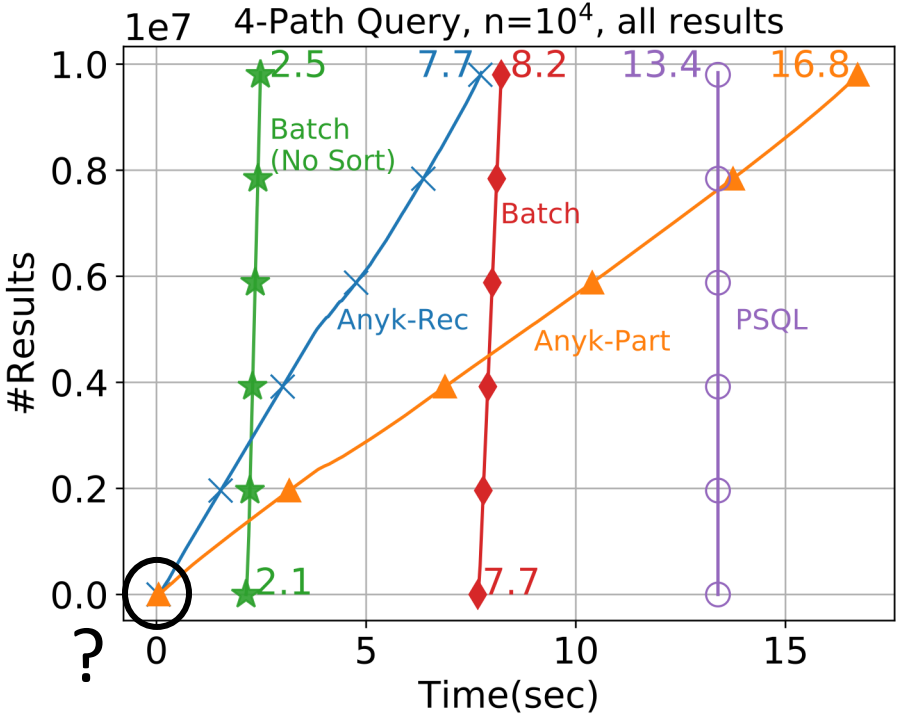


- Trees → Cycles
  - Decompose into a union of acyclic queries
  - e.g. 6-cycle TTF $= O(n^{5/3})$
    same as state-of-the-art Boolean query

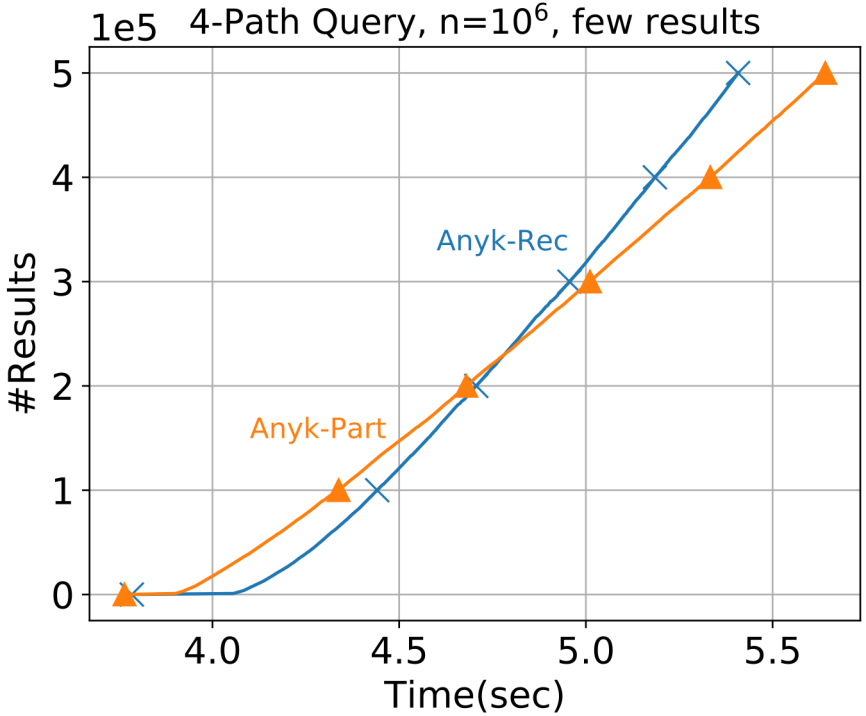
- Ranking Function besides minimum sum of weights? $(\min, +)$
  - $(\min, \max)$: min traffic congestion
  - $(\max, \times)$ for non-negative reals: highest-prob. results
  - Lexicographic ordering (any, independent of join order)

  Algebraic characterization as selective dioids

Experiments



- Anyk starts much faster than Batch
- Anyk-Rec also finishes faster than Batch



- Anyk-Part is usually faster in the beginning

Conclusions

- Ranked enumeration of arbitrary conjunctive queries
  [Yang+ ExploreDB'18]
  - Linear pre-processing (or higher for cyclic)
  - Logarithmic delay
  - Two competing algorithmic approaches

- Acknowledgements
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